CALL'S

DECIMAL ARITHMETIC,

ON A

NEW AND IMPROVED PLAN THROUGHOUT,

COMPRISING

SEVERAL NEW METHODS

OF

COMPUTING SIMPLE INTEREST, DEODECIMALS,

AND

GREAT IMPROVEMENT IN MULTIPLICATION AND DIVISION

NOT FOUND IN ANY OTHER WORK EXTANT.

BY OSMAN CALL.

L. A. FLETCHER,
HANCOCK FACTORY, N. H.
1842.

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PREFACE.

In presenting to the public a new School-Book, it may be proper to state the circumstances which have led to its publication.

The author, Mr. *Osman Call*, having prepared manuscripts on Reduction, Reduction of Currencies, Decimals, Interest, and Square and Cubic Measure, in which he had given new rules and new methods of performing arithmetical questions, wished to present his system to the public.

Knowing that a complete system of arithmetic would be required, in order to render his book useful and acceptable to the public, he engaged the subscriber to act as editor in preparing a book for the press, in which his peculiar method should be found. In engaging in this enterprise, we felt our incompetency to do justice to the science of arithmetic. And the farther we proceeded, the more clearly did we see the wide field, which we had not explored, rising to our vision.

But unlike many other subjects, arithmetic cannot lead one far astray, if a close adherence be maintained to the exact course indicated by *figures*. Hence, we have ventured to depart from the beaten path of arithmeticians, and mark out a new arrangement, new rules, and a new method of operation peculiar to this work.

The subscriber made use of Mr. Call's manuscripts in preparing those articles in which his peculiar method is given. He made such selections from these manuscripts as he thought proper; added such new matter as, in his opinion, would be useful to the

student, and labored to make that portion of the work as intelligible as possible.

Throughout other parts of the work, the subscriber has made his own selections and arrangement, and readily assumes whatever responsibility rests upon an author. He has not confined himself to the "old method" in any article in the work. We have depended mostly upon our own resources for rules and illustrations throughout the book; yet we have not hesitated to insert questions found in other books, whenever we thought proper.

The following authors have been consulted in our preparation, from each of whom we have derived greater or less aid: Pike, Lacroix, Stevens, Montgomery, Greenleaf, Adams, Little, Walsh, Euler, Allen, Day, American Encyclopedia, and Edinburgh Encyclopedia. We have aimed to make our book a *useful* school-book; how far we have succeeded, we shall leave to the public to judge. The Editor feels it due to himself to state, that, residing at so great a distance from the press, and thus being unable to review his proof sheets, many more errors may be found in the work, than would otherwise have escaped notice.

The attention of teachers, and others, is requested in relation to points of improvement, any suggestions from whom will be thankfully received by the Editor, that any errors that may occur in this edition may be corrected in the next, and such other new matter inserted, as shall be useful in advancing the knowledge of arithmetic.

ZEBULON JONES,
EDITOR.

Peterborough, N. H., Jan. 3, 1842.

among some Eastern nations, was introduced about the second century of the Christian era, and was considered as the invention of Ptolemy. In this system, every unit was divided into sixty parts, and each of these parts was divided into sixty other parts. Sixty was sexagesima prima, and was represented thus, I′; twice 60 was represented thus, II′, and thrice 60 thus, III′. This method was pursued till the series closed with 59 times 60; when the second series was commenced with 60 times 60, marked thus, I″. A dash at the bottom, thus, I,, or at the top, thus, ′I, denoted $\frac{1}{60}$.

It remained to a fierce and uncivilized people, in the eighth century, to develop a system of notation, which supplanted the sexagesimal arithmetic, and has been perpetuated to our time, as the most perfect system ever invented. The Arabs expressed numbers by proper signs, and made the value of a figure depend upon its location, but with reference also to its primitive value. To the Arabians we are indebted for our present system of Arithmetic, although it is affirmed that they regarded it to be of Indian origin. The first treatise on decimals was written in 1582; and since that time, decimal fractions have been considered essential to every system of Arithmetic.

CASUAL REMARKS.

FIGURES being characters used to express numbers and parts of numbers, and representing any thing we have occasion to deal in, and which cannot be understood without some other character or language, to give them a name, it will be readily seen that they may represent money, or any specific article or articles, with propriety. Therefore all things may be reckoned the same as if they were money; and in any arithmetical question, every kind of commodity may be treated, in whole numbers or in parts, as if of one denomination; and by learning to reckon money, and comparing the whole numbers and parts (first calling it money) with other things and the same parts of things, we readily see that all denominations are reckoned exactly alike, which gives the learner an understanding of all varieties of business in a simple and easy manner at once. By this method of computation, calling every thing money, any one may, in a very short time, understand the science, so as to be expert in every kind of business in which figures are used.

It is necessary to be partially acquainted with the old system, before entering upon the improved method; for which purpose, exercises are given for the learner before he enters upon the peculiarities of the improved system.

In all the fundamental rules, decimals are employed, and the rules given, without the word *decimal* being used; for in the sequel of the work a more minute explanation is given. Those who have but a short time to attend school, and have some previous knowledge of the science, will do well to commence at DECIMAL FRACTIONS, page 70, and by making themselves familiar with the Tables, they may soon obtain a good practical knowledge of the science, which is better than silver and gold; for it is a treasure they can keep as long as they retain their understandings.

CONTENTS.

ARITHMETIC.

ARITHMETIC teaches to compute numbers by means of certain signs and characters. The word Arithmetic is derived from the Greek *arithmos*, which means *number*. The fundamental rules of Arithmetic are Notation, Enumeration, Addition, Subtraction, Multiplication, and Division.

The scholar should become thoroughly acquainted with these rules before leaving them.

I. Notation.

Notation is the method of expressing numbers by characters or figures. Different kinds of notation have been employed by different nations. The Greeks and Romans employed the letters of their alphabets to express numbers. The Arabic method of notation employs these ten characters,—1 one, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine, 0 cipher. It will be necessary to consider, in this work, only the Roman and Arabic methods of expressing numbers.

1. ROMAN METHOD OF NOTATION.

In this method, seven letters are used to express numbers, viz. I, V, X, L, C, D, M; I denoting 1; V, 5; X, 10; L, 50; C, 100; D, 500; and M, 1000. IƆ are sometimes used for D, and CIƆ for M.

2

TABLE.

I ...1	X 10	C 100	M........1000	$\overline{X}$.. 10000
II ...2	XX....20	CC 200	MM or $\overline{II}$.2000	$\overline{XX}$ 20000
III ..3	XXX ..30	CCC ...300	$\overline{III}$ 3000	$\overline{XXX}$.....30000
IV ..4	XL....40	CCCC ..400	$\overline{IV}$ 4000	$\overline{XXXX}$....40000
V ...5	L 50	D 500	$\overline{V}$ 5000	IↃↃↃↃ ...50000
VI ..6	LX ...60	DC 600	$\overline{VI}$.......6000	$\overline{C}$........100000
VII . 7	LXX ..70	DCC....700	$\overline{VII}$......7000	$\overline{M}$ 1000000
VIII 8	LXXX.80	DCCC ..800	$\overline{VIII}$.....8000	$\overline{MM}$ 2000000
IX ..9	XC ...90	DCCCC.900	$\overline{IX}$ 9000	

2. ARABIC METHOD OF NOTATION.

In this method of notation, the following characters are employed, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. These characters possess two values, *primitive* and *local.* The *primitive* value of a figure is that which it has when compared with another of the *same* denomination. Thus, 1, 2, 3, 4, being units, 2 has twice the value of 1; 3 has three times the value of 1, &c. If 1, 2, 3, 4, are considered any other denomination than units, but are all of the *same* kind, then 2 has twice the value of 1; 3 has three times the value of 1, as before.

The *local* value of a figure is that which it has, when compared with another figure of a *different* denomination. Thus, 1, 1, 1, taken as different denominations, have three different values, according to their locality. The local value of figures increases from the right hand to the left, in a ten-fold proportion. If we wish to express the number of days in a year, in Arabic characters, thus, 365, the first right-hand figure, 5, represents five days; the next left-hand figure, 6, represents sixty days; and the figure 3 represents three hundred days. If we wish to express in figures, one dollar, twenty-five cents, and five mills, thus, 1,255, the right-hand figure 5 represents five mills; the next left-hand figure, 5, represents five cents; the next left-hand figure, 2, represents twenty cents; and the figure 1, one hundred cents, or one dollar. On the other hand, the local value of figures diminishes in a ten-fold proportion, from the left hand to the right.

II. Enumeration.

Enumeration teaches to express in language the value of figures, by ascertaining the local value of each figure in the given sum. The principles of Enumeration may be best learned from the following

TABLE.

5389746251768534789,386215625468

5	Trillions, &c.
3	Hundreds of thousands of billions
8	Tens of thousands of billions
9	Thousands of billions
7	Hundreds of billions
4	Tens of billions
6	Billions
2	Hundreds of thousands of millions
5	Tens of thousands of millions
1	Thousands of millions
7	Hundreds of millions
6	Tens of millions
8	Millions
5	Hundreds of thousands
3	Tens of thousands
4	Thousands
7	Hundreds
8	Tens
9	Units
3	Tenths
8	Hundredths
6	Thousandths
2	Tenths of thousandths
1	Hundredths of thousandths
5	Millionths
6	Tenths of millionths
2	Hundredths of millionths
5	Thousandths of millionths
4	Tenths of thousandths of millionths
6	Hundredths of thousandths of millionths
8	Billionths, &c.

Should the learner find it necessary to employ a larger number of figures than are given in the preceding table, he can proceed with trillions, as he did with billions, making use of quadrillion, quintillion, sextillion, septillion, octillion, nonillion, decillion, undecillion, duodecillion, &c.

III. Addition.

ADDITION IS UNITING TWO OR MORE NUMBERS INTO ONE SUM.

RULE.—*Write the numbers to be added under one another, so that units, tens, hundreds, tenths, hundredths, and thousandths, &c., may be respectively under one another; that is, so that those of the same local value may be under each other. Draw a line under the whole; then, beginning at the right*

hand column, add them, one after another. If the sum be less than 10, *write it under the column added; if it be* 10 *or more, set down the right-hand character or figure, and add the left-hand figure to the next column. Observe the same rule with each column, and at the last column write the whole amount.*

Proof.—Begin at the top of the right-hand column, and reckon all the figures downward; if the amount agree with the answer, the work may be supposed to be right.

Signs.—The sign of addition is a short, horizontal line, crossed by a perpendicular, thus, +, and shows that a number placed before it, is to be added to a number placed after it. Two parallel horizontal lines, thus, =, are the sign of equality; as, 8+4=12.

Ex. 1.—	47,5
	150,75
	37,375
Ans.	235,625

In this example, three numbers are given to be united into one sum. Begin at the right-hand figure, and say, five are five, and place the 5 directly under the figure 5. Then say, seven and five are twelve, and place 2 directly under 7, and add 1 to the next left-hand column. Proceed in this manner to the left-hand column, where you must write the whole amount.

Ex. 2.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Tenths of thousandths.
	1	1	2	5,	3	7	5	
		3	7	5,	2	5		
			4	9,	0	6	2	5
			8	3,	1	8	7	5
Ans.	1	6	3	2,	8	7	5	0

Ex. 3.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Tenths of thousandths.
	4	3,	7	5		
		8,	9	3	7	5
	3	6,	8	1	2	5
Ans.	8	9,	5	0	0	0

4. A man sold a horse for seventy-five dollars; a saddle for nine dollars and seventy-five cents; a bridle for one dollar, thirty-seven and a half cents. How much money did he receive? *Ans.* $96,125.

5. A man bought a lot of land for ninety-seven dollars, eighty-seven cents and five mills; a yoke of oxen for one hundred and twelve dollars; a horse for seventy-nine dollars and ninety cents; a wagon for thirty-two dollars and twenty-five cents; and a harness for nineteen dollars, sixty-two and a half cents. How much money did he pay? *Ans.* $337,65.

IV. Subtraction.

Subtraction is taking a less number from a greater. The greater number is called the *minuend,* the less number is called the *subtrahend,* and the result is called *difference,* or *remainder.*

RULE.—*Write the numbers according to their local value, units under units, tens under tens, hundreds under hundreds, tenths under tenths, hundredths under hundredths, &c. Draw a line under them, and beginning at the right hand, subtract the figure of the subtrahend from the figure above it in the minuend, and write the difference below. If there be no figure in the minuend, from which to subtract the figure in the subtrahend, or, if the figure in the minuend be less than the figure below it in the subtrahend, suppose* 10 *to be added to the minuend, and from that amount subtract, observing to add* 1 *to the next left-hand figure in the subtrahend before subtracting.*

PROOF.—Add the remainder to the subtrahend, and if the amount agree with the minuend, the work may be supposed to be right.

SIGN.—A short, horizontal line, thus, —, placed between

two numbers, shows that the number at the right hand is to be subtracted from the number at the left. As, 8—6=2.

Ex. 1 17,25 minuend.
14,375 subtrahend.
2,875 remainder.

In this example, I place the larger number, 17,25, uppermost, and call it the minuend. Under the minuend, I write the smaller number, 14,375, calling it the subtrahend. Beginning at the right hand, I subtract 5 from 10, according to the rule, and add 1 to 7 before subtracting it. I subtract 8 from 15, and add 1 to 3 before subtracting it. I subtract 4 from 12, and add 1 to 4 before subtracting it. To prove this sum, add the remainder to the subtrahend, and the amount will agree with the minuend.

2. From 36,375 take 25,625.

OPERATION.

36,375 minuend.
25,625 subtrahend.

10,750 remainder.

3. From $50,375 take $35,625.

50,375
35,625

14,750 rem.

4. From 3000009,76 take 4078,37. *Ans.* 2995931,39.

5. From 7,00009 take 5,345605. *Ans.* 1,654485.

6. From thirty-five thousands take thirty-five thousandths. *Ans.* 34999,965.

7. From 21 take 19,777. *Ans.* 1,223.

8. From 8 and 8 hundredths take 3 and 3 tenths. *Ans.* 4,78.

9. A man bought a horse for 60 dollars; a wagon for 35 dollars and 75 cents. How much more did he give for the horse than the wagon? *Ans.* 24,25.

V. Multiplication.

"When a number is added to itself once, it is said to be doubled; when it is added to itself twice, it is said to be tripled; or, in general, the operation of adding a number to itself a certain number of times, is called *multiplication.* The number which is added to itself is called the *multiplicand,* and the one which expresses the number of additions, the *multiplier;* both are called *factors.*"* The result, or answer, is called the *product.*

SIGN.—The sign of multiplication is two lines crossing each other in the form of the letter X, and shows that the number at the left hand of the cross is to be multiplied by the number at the right hand of it. As, $8 \times 4 = 32$.

The scholar should now make himself familiar with the following

MULTIPLICATION TABLE.

2 times	3 times	4 times	5 times	6 times	7 times	8 times	9 times	10 times	11 times	12 times
1= 2	1= 3	1= 4	1= 5	1= 6	1= 7	1= 8	1= 9	1= 10	1= 11	1= 12
2= 4	2= 6	2= 8	2=10	2=12	2=14	2=16	2= 18	2= 20	2= 22	2= 24
3= 6	3= 9	3=12	3=15	3=18	3=21	3=24	3= 27	3= 30	3= 33	3= 36
4= 8	4=12	4=16	4=20	4=24	4=28	4=32	4= 36	4= 40	4= 44	4= 48
5=10	5=15	5=20	5=25	5=30	5=35	5=40	5= 45	5= 50	5= 55	5= 60
6=12	6=18	6=24	6=30	6=36	6=42	6=48	6= 54	6= 60	6= 66	6= 72
7=14	7=21	7=28	7=35	7=42	7=49	7=56	7= 63	7= 70	7= 77	7= 84
8=16	8=24	8=32	8=40	8=48	8=56	8=64	8= 72	8= 80	8= 88	8= 96
9=18	9=27	9=36	9=45	9=54	9=63	9=72	9= 81	9= 90	9= 99	9=108
10=20	10=30	10=40	10=50	10=60	10=70	10=80	10= 90	10=100	10=110	10=120
11=22	11=33	11=44	11=55	11=66	11=77	11=88	11= 99	11=110	11=121	11=132
12=24	12=36	12=48	12=60	12=72	12=84	12=96	12=108	12=120	12=132	12=144

RULE.—*Place the multiplier under the multiplicand, without regard to the local value of the figures. Beginning at the right hand, multiply each figure in the multiplicand by each figure in the multiplier, and place the first figure of each product directly under the multiplying figure, setting down the right-hand figure, and adding the left-hand figure to the next figure in the product. Having gone through in this manner with all the figures in the multiplier, add their several products, and cut off as many figures from the right hand as there are decimal places in both factors.*

* Ed. Ency. *in loco.*

Ex. 1. Multiply 82 by 12.

```
         82
         12
        ---
        164
        82
        ---
Product. 984
```

In this example, having set down the figures according to the rule, I begin with the right-hand figure in the multiplier, and by it multiply each figure in the multiplicand, setting down the first figure of the product under 2 in the multiplier. I then begin with the next figure in the multiplier, and multiply each figure in the multiplicand by it, placing the first figure of the product under the multiplying figure. Having multiplied by both figures in the multiplier, I add the two products, and their sum is the required product, or answer.

Ex. 2. Multiply 18,25 by 6,25.

```
            18,25
             6,25
           ------
             9125
            3650
          10950
          --------
Product.  114,0625
```

Ex. 3. A man bought 8 bushels and a half of wheat, at one dollar and a half per bushel; how much did he give for the wheat?

```
           1,5
           8,5
           ---
           7 5
          120
          -----
Ans.    $12,75
```

4. A man sold six calves for five dollars and fifty cents each; how much did he obtain for the calves? *Ans.* $33.

5. Multiply ,09875 by ,0625. *Ans.* ,006171875.

6. Multiply ,99999 by ,99999. *Ans.* ,9999800001.

7. Multiply ,5 by ,5. *Ans.* ,25.

8. Multiply ,25 by ,25. *Ans.* ,0625.

When there are not so many figures in the product as there are decimals in the factors, prefix a cipher or ciphers, until there be as many figures in the product as there are decimals in the factors, and point off as before directed.

9. Multiply six and one fourth cents by the same.
Ans. ,00390625.

A composite number is one which can be measured exactly by another number exceeding unity. Thus, 6 may be divided by 3 or 2. Hence, 2 and 3 are the component parts of 6. The number 15 may be divided by 3 or 5; therefore, 3 and 5 are the component parts of 15.

When the multiplier is a composite number, the operation may be made easier, by multiplying first by one component part of the multiplier, and that product by the other.

10. Multiply 89 by 49. *Ans.* 4361.

When the multiplier is 1, with any number of ciphers annexed, the multiplication may be performed by annexing as many ciphers to the multiplicand as there are ciphers in the multiplier.

11. Multiply 89 by 100. *Ans.* 8900.
12. Multiply 1728 by 100000. *Ans.* 172800000.

Multiplication may be proved by casting out the nines, or threes, out of the multiplicand, multiplier, and product.

EXAMPLE.

Multiply 17249 by 1725.

```
   17249
    1725
   -----
   86245
  34498
 120743
 17249
--------
29754525
```

```
  3
6 X 5
  3
```

Having performed the multiplication, begin with the multiplicand, and add the figures from the right hand to the

left, dropping the nines, and place the excess at the right hand of the cross. Cast out the nines of the multiplier, and place the excess at the left hand of the cross. Multiply these two figures together, cast out the nines of their product, and place the excess over the product. Finally, cast out the nines of the product, and place the excess under the cross; and if the figures at the top and bottom agree, the work is supposed to be right.

Multiplication may be proved by division, addition, and subtraction. To prove multiplication by division, divide the product by the multiplier, and the quotient will be the multiplicand. To prove multiplication by addition, set down the multiplicand as many times as there are units in the multiplier, and add their several numbers, and their amount will agree with the required product. If the multiplier be a fraction, set down such a part of the multiplicand as is denoted by the multiplier. To prove multiplication by subtraction, subtract the multiplicand from the product as many times as there are units in the multiplier; if the work is right, there will be no remainder.

If the multiplier be a fraction, take such a part of the multiplicand from the product as is denoted by the multiplier.

EXAMPLES.

$8 \times 4 = 32$. Proof by division. $32 \div 4 = 8$.
Proof by addition. $8+8+8+8=32$.
Proof by subtraction. $32-8=24-8=16-8=8-8=0$.

Hence it is obvious that multiplication is a method of repeating a given number a certain number of times. This repetition may be made by successive additions, and may therefore be proved by addition and subtraction, as well as division, and casting out the nines.

VI. Division.

Division consists in finding one factor, when the other, with the product, is given. Multiplication may be performed, as we have seen, by successive additions; in like

manner, division may be performed by repeated subtractions of the divisor from the dividend. The number to be divided is called the *dividend*, the given factor is called the *divisor*, and the factor sought, is called the *quotient*.

SIGN.—The sign of division is a short, horizontal line between two dots, thus, ÷, and shows that the number at the left hand of the line is to be divided by the number at the right hand; thus, 8÷4=2.

RULE.—*Write the divisor at the right or left hand of the dividend, and draw a vertical line between it and the dividend. Seek how many times the divisor is contained in the first left-hand figure, or figures, of the dividend. Write the figure, showing how many times the divisor is contained in the first left-hand figure, or figures, of the dividend, on the right-hand side of the dividend, under the divisor, calling it the quotient figure. Multiply the divisor by the quotient figure, and place the product under the left-hand figure, or figures, of the dividend in which the divisor was contained once, or more. Subtract this product from the figures above it, and to the right hand of the remainder bring down the next undivided figure of the dividend, and proceed as before directed. Proceed in this manner till all the figures of the dividend be divided. If there be a remainder, ciphers may be added, and the division be continued to an indefinite extent, or till there be no remainder. Point off as many figures for decimals from the right hand of the quotient as the decimals in the dividend exceed those in the divisor; that is, the decimals in the divisor and quotient must equal the number of decimals in the dividend.*

Ex. 1.—Divide 7956 by 6.

```
Dividend. 7956 | 6 divisor.
          6    | 1326 quotient.
          —
          19
          18
          ——
           15
           12
           ——
            36
            36
            ——
```

2. Divide 738652 by 9. *Ans.* 82072, and 4 rem.
3. Divide 786525 by 75. *Ans.* 10487.
4. Divide 4,25 by ,125. *Ans.* 34.
5. Divide 37865 by 6,25. *Ans.* 6058,4.
6. Divide ,012 by ,005. *Ans.* 2,4.
7. Divide 1, by ,0001. *Ans.* 1000.
8. Divide ,5 by ,000005. *Ans.* 100000.

When the divisor consists of one or more figures with ciphers annexed, cut off the ciphers and as many figures on the right hand of the dividend, and proceed with the remaining figures, as in the foregoing examples. The figures cut off in the dividend must be annexed to the remainder.

9. Divide 8765256 by 500. *Ans.* 17530.
Rem. 256.

When the divisor consists of only one figure, or of a number less than 12, a part of the process may be carried on in the mind, and the process shortened.

10. Divide 7956 by 6.

$$\begin{array}{r|l} 6 & 7956 \\ \hline & 1326 \text{ quotient.} \end{array}$$

In this example I find that 6 are contained in 7 once, and there is 1 remainder. This 1 remainder, being a part of 7, retains the local value of 7, or thousands, and when taken with 9 hundreds, makes 19 hundreds. I then seek how many times 6 are contained in 19; I find the quotient figure to be 3, and there is 1 remainder. This 1 remainder is a part of the 19 hundreds, and retains the local value of hundreds, and, when taken with 5 tens, makes 15 tens. I then seek how many times 6 are contained in 15; and find the quotient to be 2, and there are 3 remainder. This figure, 3, retains the value of tens, and, when taken with 6 units, makes 36 units. I then find that 6 are contained in 36 6 times.

11. Divide 738652 by 9. *Ans.* 82072, and 4 rem.

When the divisor is a composite number, the process may be varied by dividing first by one component part, and that quotient by the other.

12. Divide 576 by 48.

$$\begin{array}{r|l} 6 & 576 \\ \hline 8 & 96 \\ \hline \end{array}$$

Ans. 12

13. Divide 1260 by 63.

$$\begin{array}{r|l} 7 & 1260 \\ \hline 9 & 180 \\ \hline \end{array}$$

Ans. 20

14. Divide 2430 by 81.

15. Divide 448 by 56.

When the divisor is 10, 100, 1000, &c., cut off from the right hand of the dividend as many figures as there are ciphers in the divisor; the figures on the left of the point will be the quotient, and the figures on the right of the point will be the remainder.

16. Divide 8756 by 100. *Ans.* 87,56

17. Divide 54321 by 1000. *Ans.* 54,321.

To prove division, multiply the quotient by the divisor, and to that product add the remainder, if there be any; and if the work is right, the product will agree with the dividend.

VII. Fractions.*

Fractions are of two kinds, VULGAR and DECIMAL. The rules for *Decimals* will be given in another part of this work; therefore, it will be necessary, in this place, to consider only

VULGAR FRACTIONS.

A fraction is a part of a unit. Thus, 1 is a unit; $\frac{1}{2}$ is a part of 1, and is, therefore, a fraction. Fractions arise from division. If we divide 13 by 5, thus, 5 | 13 | $2\frac{3}{5}$, the quotient will be 2, and the remainder 3. This remainder is an undivided part of the *dividend* 13; and, as 5 are not contained in 3, the division of 3 may be expressed by writ-

* This article on FRACTIONS was prepared wholly by the Editor.

ing 5 under it, thus, $\frac{3}{5}$. The whole quotient of 13 divided by 5, is $2\frac{3}{5}$. From these remarks it appears, that in the fraction $\frac{3}{5}$, the upper number is to be divided, or is the dividend, being an undivided part of the dividend 13. The lower number is the divisor. The upper number, 3, is also called the *numerator*, and the lower number the *denominator*. The propriety of these terms will appear when we take another view of a fractional expression. Let an apple be divided into five equal parts, one part would be expressed thus, $\frac{1}{5}$; two parts thus, $\frac{2}{5}$; three parts thus, $\frac{3}{5}$; four parts thus, $\frac{4}{5}$. From these expressions it is shown, that the figure representing the number of parts of the unit, taken in any example, is always made the *numerator* in a fractional expression. Thus three parts of the apple are represented in this manner, $\frac{3}{5}$. This expression shows that three parts of the apple are taken. The *numerator*, then, *numbers* the parts of the unit in any fraction; and the meaning of the word is, *one that numbers, or counts.* From the same expressions it appears that the denominator shows into how many parts the unit is divided. When an apple is divided into five equal parts, the numeral 5 is made the *denominator.* And universally, the numeral expressing the number of parts into which the unit is divided, is made the denominator. This numeral names the fraction; and the meaning of denominator is *one that gives a name, or namer.*

In reading fractional expressions, the denominator is considered an *ordinal*, and not a *numeral.* Thus, $\frac{1}{5}$ is read, one fifth, and not, one five; $\frac{2}{5}$ is read, two fifths, and not, two fives. And universally, in reading vulgar fractions, the *ordinal* of the denominating numeral is employed. Thus, $\frac{1}{3}$ is read, one third; $\frac{1}{4}$ is read, one fourth; $\frac{1}{6}$ is read, one sixth; $\frac{1}{7}$ is read, one seventh.

The learner should now attend carefully to the following

REMARKS.

1. Fractions are parts of a unit.
2. When the numerator of a fraction is less than the denominator, thus, $\frac{1}{3}$, the fraction is called a *proper fraction;* when the numerator is not less than the denominator, thus, $\frac{3}{3}$, or $\frac{4}{3}$, the expression is called an *improper fraction.*

3. When a whole number and a fraction are written, thus, $2\frac{1}{5}$, the expression is called a *mixed number*.

4. A *simple fraction* has one numerator and one denominator in the same expression; thus, $\frac{1}{2}$, $\frac{1}{4}$.

5. A *compound fraction* is a fraction of a fraction, and may be known by the word *of* placed between the two simple fractions; thus, $\frac{1}{2}$ of $\frac{3}{4}$.

I.

TO REDUCE A FRACTION TO ITS LOWEST TERMS.

Divide the numerator and denominator by any number that will divide them both without a remainder, and that quotient by any number that will divide it without a remainder, and so on, till no number, exceeding unity, will divide the fraction without a remainder.

1. Reduce $\frac{15}{175}$ to its lowest terms.

OPERATION.

$$\frac{15}{175} \,|\, 5 = \frac{3}{35}, \text{ } \textit{Ans.}$$

NOTE.—The value of a fraction is not altered, unless the ratio of the numerator to the denominator is changed. But in the above example, the ratio which the numerator has to the denominator, is not changed; therefore the value of the fraction is not altered. The numerator, 15, is the same multiple of 3, that the denominator, 175, is of 35. Therefore, as $\frac{3}{35}$ have the same ratio as $\frac{15}{175}$, the values of the two expressions are the same.

2. Reduce $\frac{225}{1125}$ to its lowest terms. *Ans.* $\frac{1}{5}$.
3. Reduce $\frac{342}{855}$ to its lowest terms. *Ans.* $\frac{2}{5}$.
4. Reduce $\frac{1536}{4608}$ to its lowest terms. *Ans.* $\frac{1}{3}$.
5. Reduce $\frac{72}{432}$ to its lowest terms. *Ans.* $\frac{1}{6}$.

The process of finding the lowest terms of a fraction may be varied by finding the greatest common divisor of that fraction.

1. Reduce $\frac{96}{544}$ to its lowest terms.

OPERATION.

```
96|544|5
   480
   ———
    64|96|1
       64
       —
       32|64|2
          64
          —
```

By this process, it is found that 32 are the greatest common divisor of 96 and 544. Therefore the fraction, reduced to its lowest terms, is $\frac{96}{544} | 32 = | \frac{3}{17}$, *Ans.*

NOTE.—By inspecting the above process, it will be seen that 32 are the measure of 64, the first remainder, and 96. Therefore they will be the measure of $\overline{96 \times 5} + 64$. As 32 are the measure of 96, they will be the measure of 96, multiplied by any number, and that product increased by 32 taken any number of times.

Thus, 32 are the measure of $\overline{96 \times 2} + 32$.
" " " $\overline{96 \times 3} + 64$.
" " " $\overline{96 \times 4} + 64$.
" " " $\overline{96 \times 5} + 64$.
" " " $\overline{96 \times 6} + 128$.
" " " $\overline{96 \times 7} + 256$, &c.

And universally, any number that is a measure of the remainder and divisor will be a measure of the dividend; and also of the divisor multiplied by any number, and that product increased by the remainder taken any number of times.

2. Reduce $\frac{24}{84}$ to its lowest terms. *Ans.* $\frac{2}{7}$.
3. Reduce $\frac{25}{175}$ to its lowest terms. *Ans.* $\frac{1}{7}$.
4. Reduce $\frac{268}{1896}$ to its lowest terms. *Ans.* $\frac{67}{474}$.

II.

TO REDUCE A MIXED NUMBER TO AN IMPROPER FRACTION.

Multiply the whole number by the denominator, and to the product add the numerator; the sum will be the numerator of the improper fraction, and the denominator will be the same as before.

1. Reduce $13\frac{5}{6}$ to an improper fraction.

OPERATION.

$$\begin{array}{l} 13\frac{5}{6} \\ 6 \\ \hline \frac{83}{6}=\frac{83}{6},\text{* } Ans. \end{array}$$

2. Reduce $19\frac{7}{8}$ to an improper fraction. *Ans.* $\frac{159}{8}$.
3. Reduce $6\frac{3}{4}$ to an improper fraction. *Ans.* $\frac{27}{4}$.

III.

TO REDUCE AN IMPROPER FRACTION TO A WHOLE, OR MIXED NUMBER.

Divide the numerator by the denominator, and the quotient will be the whole number. If there be a remainder, write it over the divisor, at the right hand of the quotient.

1. Reduce $\frac{189}{4}$ to a whole, or mixed number.

OPERATION.

$$\begin{array}{l} 4\,|\,189 \\ \hline \quad 47\frac{1}{4},\ Ans. \end{array}$$

In this example, the unit is divided into fourths. Every 4 fourths make a unit. Therefore, as many times as 4 are contained in 189, so many units there will be. I find that 4 are contained in 189, 47 times, and there is 1 remainder. There are, then, in $\frac{189}{4}$, 47 units and $\frac{1}{4}$ of a unit.

2. Reduce $\frac{15324}{123}$ to a mixed number. *Ans.* $124\frac{72}{123}$
3. Reduce $\frac{1148}{61}$ to a mixed number. *Ans.* $18\frac{50}{61}$
4. Reduce $\frac{1161}{617}$ to a mixed number. *Ans.* $1\frac{544}{617}$
5. Reduce $\frac{9999}{100}$ to a mixed number. *Ans.* $99\frac{99}{100}$
6. Reduce $\frac{1000000}{1}$ to a whole number. *Ans.* 1000000.

* This process consists simply in ascertaining the number of sixths in 13 and $\frac{5}{6}$. In one unit there are six sixths, and in 13 units there are thirteen times as many sixths as in 1 unit. Hence, 13×6, or 78, express the number of sixths in 13. To 78 add 5, because in $\frac{5}{6}$ there are 5 sixths, and the amount, 83, expresses the number of sixths in $13\frac{5}{6}$.

IV.

TO REDUCE A COMPOUND FRACTION TO A SIMPLE ONE.

Multiply the numerators together for a new numerator, and the denominators for a new denominator. If there be mixed numbers, reduce them to improper fractions, and proceed as the rule directs.

1. Reduce $\frac{1}{2}$ of $\frac{7}{8}$ of $\frac{9}{10}$ to a simple fraction.

OPERATION.

$$1\times7\times9=63$$
$$2\times8\times10=\overline{160}$$

The propriety of this process may be seen from the following analysis. One eighth of $\frac{9}{10}$ is nine eightieths; $\frac{7}{8}$ of $\frac{9}{10}$ are seven times as much as one eighth of $\frac{9}{10}$; and seven times $\frac{9}{80}$ are $\frac{63}{80}$. $\frac{1}{2}$ of $\frac{63}{80}$ is $\frac{63}{160}$, *Ans.*

2. Reduce $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{11}{12}$ to a simple fraction.
Ans. $\frac{385}{864}$.

3. Reduce $\frac{6}{7}$ of $\frac{11}{15}$ of $\frac{3}{5}$ of $\frac{1}{13}$ to a simple fraction.
Ans. $\frac{11}{525}$.

4. Reduce $\frac{3}{5}$ of $\frac{9}{17}$ of 265 to a mixed number.
Ans. $84\frac{3}{17}$.

5. Reduce $7\frac{1}{2}$ of $3\frac{1}{5}$ of 8041 to a whole number.
Ans. 192984.

6. Reduce $\dfrac{7\frac{1}{7}}{8\frac{1}{3}}$ of $\dfrac{6\frac{1}{9}}{10\frac{2}{11}}$ of $9\frac{1}{13}$ to a mixed number.

OPERATION.

$$\frac{7\frac{1}{7}}{8\frac{1}{3}}\times7=\frac{50}{58\frac{1}{3}}\times3=\frac{150}{175}=\frac{7\frac{1}{7}}{8\frac{1}{3}}$$

$$\frac{6\frac{1}{9}}{10\frac{2}{11}}9\times=\frac{55}{91\frac{7}{11}}\times11=\frac{605}{1008}=\frac{6\frac{1}{9}}{10\frac{2}{11}}$$

$$\begin{array}{r} 9\frac{1}{13} \\ 13 \\ \hline 118 \\ \hline 13 \end{array}$$

Hence the compound fraction now is,

$$\frac{150}{175} \text{ of } \frac{605}{1008} \text{ of } \frac{118}{13}=4\frac{15357}{22932}, \textit{ Ans.}$$

The first part of this process consists in clearing the numerators and denominators of fractions. This is done by multiplying all the terms of the mixed fraction by the denominators. It is plain that the value of the expression is not changed in doing this, because all the terms are multiplied by the same number.

7. Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5\frac{1}{3}}{8\frac{1}{9}}$ to a simple fraction. *Ans.* $\frac{18}{73}$.

8. Reduce $\frac{1}{3}$ of $\frac{1}{6}$ of $\frac{1}{9}$ to a simple fraction. *Ans.* $\frac{1}{162}$.

V

TO MULTIPLY A WHOLE NUMBER BY A FRACTION.

Multiply the whole number by the numerator, and divide by the denominator; or divide by the denominator, and multiply by the numerator.

1. Multiply 87 by $\frac{3}{4}$.

OPERATION.

$$\begin{array}{r|l} & 87 \\ & \underline{\quad 3} \\ 4 & \underline{261} \\ & 65\frac{1}{4}.\ \textit{Ans.} \end{array}$$

Or thus,

$$\begin{array}{r|l} 4 & \underline{87} \\ & 21\frac{3}{4} \\ & \underline{\quad 3} \\ & 65\frac{1}{4} \end{array}$$

The last process may be analyzed thus: one fourth of 87 is $21\frac{3}{4}$; and $\frac{3}{4}$ of 87 are 3 times $21\frac{3}{4}$, or $65\frac{1}{4}$.

The first process amounts to this: one fourth of 261 is equal to $\frac{3}{4}$ of 87. As $\frac{3}{4}$ of 1 are equal to $1\times3\div4=\frac{3}{4}$; or $1\div4\times3=\frac{3}{4}$; so $87\times3\div4=65\frac{1}{4}$; or $87\div4\times3=65\frac{1}{4}$.

2. Multiply 8756 by $\frac{9}{11}$. *Ans.* 7164.
3. Multiply 45 by $\frac{7}{10}$. *Ans.* $31\frac{5}{10}$.
4. Multiply 75 by $\frac{3}{7}$. *Ans.* $32\frac{1}{7}$.
5. Multiply 84 by $\frac{9}{13}$. *Ans.* $58\frac{2}{13}$.

VI.

TO DIVIDE A WHOLE NUMBER BY A FRACTION.

Multiply the whole number by the denominator of the fraction, and divide the product by the numerator.

1. Divide 6 by $\frac{1}{2}$.	2. Divide 8 by $\frac{3}{4}$.
6	8
2	4
1 | 12	3 | 32
12	$10\frac{2}{3}$

In example first, I wish to ascertain how many times $\frac{1}{2}$ is contained in 6. It is evident that it is contained in 6 as many times as there are halves in 6. Therefore, $6\times2=12$, *Ans.*

In the second example, the divisor is $\frac{3}{4}$. Had the divisor been $\frac{1}{4}$, then 8×4, or 32, would have been the quotient. But as the divisor is $\frac{3}{4}$, 32 are three times as large as the true quotient. Therefore, divide 32 by 3, and the quotient, $10\frac{2}{3}$, is the answer.

Again. $\frac{1}{4}$ is contained in 8 as many times as there are fourths in 8. And $\frac{3}{4}$ are contained in 8 as many times as there are $\frac{3}{4}$ in 8.

In 8 there are 32 fourths; as $\frac{3}{4}$ are three times as much as $\frac{1}{4}$, therefore, there will be $\frac{1}{3}$ as many three fourths as there are one fourths in 8.

Therefore, $8\times4\div3$, show how many times $\frac{3}{4}$ are contained in 8.

3. Divide 89 by $\frac{7}{11}$. *Ans.* $139\frac{6}{7}$.
4. Divide 15 by $\frac{2}{15}$. *Ans.* $112\frac{1}{2}$.
5. Divide 27 by $\frac{8}{17}$. *Ans.* $57\frac{3}{8}$.
6. Divide 128 by $\frac{1}{15}$. *Ans.* 1920.
7. Divide 98 by $\frac{1}{17}$. *Ans.* 1666.

VII.

TO MULTIPLY A FRACTION BY A WHOLE NUMBER.

Multiply the numerator by the whole number, or divide the denominator by the whole number, when this can be done without a remainder.

1. Multiply $\frac{13}{144}$ by 12.

$144 \div 12 = 12$, denominator.

$\frac{13}{12} = 1\frac{1}{12}$, *Ans.*

Or thus: $13 \times 12 = 156$, numerator.

$\frac{156}{144} = 1\frac{1}{12}$, *Ans.*

In the first example, it is plain that $\frac{13}{144}$ are multiplied by dividing the denominator by 12, because the number of parts into which the unit is divided, is diminished; and therefore their magnitude is increased. Or if the denominator be considered a divisor, then, if 144 be divided by 12, the value of the fraction is increased in the same proportion.

Again; 12 times $\frac{13}{144}$ are $\frac{156}{144}$; therefore, multiplying the numerator by 12, increases the value of the fraction twelve times.

2. Multiply $\frac{8}{9}$ by 7. *Ans.* $\frac{56}{9} = 6\frac{2}{9}$.

3. Multiply $\frac{13}{23}$ by 7. *Ans.* $3\frac{22}{23}$.

4. If one ton of hay cost \$10, what will $13\frac{13}{23}$ tons cost?

OPERATION.

$13\frac{13}{23}$
23
—
39
26
13
——
312

$\frac{312}{23} \times 10 = \frac{3120}{23} = 135\frac{15}{23}$, *Ans.*

5. If a man pays \$$2\frac{1}{2}$ for one week's board, how much must he pay for 11 weeks' board? *Ans.* $27\frac{1}{2}$.

6. If a man builds $3\frac{1}{2}$ rods of wall in one day, how many rods of wall can he build in 7 weeks, allowing that he does not work on the Sabbath? *Ans.* 147.

7. If a man pays $\frac{1}{6}$ of a mill for the use of one dollar one day, how much must he pay for the use of \$847 one day? *Ans.* ,$141\frac{1}{6}$.

VIII.

TO DIVIDE A FRACTION BY A WHOLE NUMBER.

Multiply the denominator by the whole number, or divide the numerator by the whole number, when it can be done without a remainder.

1. Divide $\frac{8}{9}$ by 8. *Ans.* $\frac{1}{9}$.
2. Divide $\frac{7}{21}$ by 7. *Ans.* $\frac{1}{21}$.

In each of these examples, the division is performed by dividing the numerator by the whole number. That $\frac{1}{9}$ is 8 times less than $\frac{8}{9}$, is plain; because the denomination of parts in the fraction remains unchanged, while the *number of parts* is diminished 8 times.

3. Divide $\frac{3}{5}$ by 5. *Ans.* $\frac{3}{5} \div 5 = \frac{3}{25}$.

In this example, the division is performed by multiplying the denominator by the whole number. That this process divides the fraction is evident, because *the parts of the fraction $\frac{3}{25}$, are 5 times less than the parts of the fraction $\frac{3}{5}$, while the number of parts is the same in both.*

4. Divide $\frac{8}{9}$ by 6. *Ans.* $\frac{8}{54}$.
5. If 13 tons of hay cost \$$144\frac{1}{3}$, what is the price of one ton? *Ans.* \$$11\frac{4}{39}$.
6. Divide $14\frac{3}{4}$ by 8. *Ans.* $1\frac{27}{32}$.
7. Divide $3\frac{4}{7}$ by 11. *Ans.* $\frac{25}{77}$.
8. Divide $8\frac{1}{3}$ by 6. *Ans.* $1\frac{7}{18}$.
9. Divide $9\frac{11}{13}$ by 5. *Ans.* $1\frac{63}{65}$.
10. Divide $10\frac{13}{23}$ by 15. *Ans.* $\frac{81}{115}$.
11. Divide $111333444\frac{123456}{347896}$ by 111876.
Ans. $995\frac{60968774}{405429301}$.
12. Divide $88990011223344557766\frac{77668855}{99885678}$ by 1243576.
Ans. $7155976894322 8\frac{85246105608119}{124215431904528}$.
13. Divide $875625375\frac{625375}{875625}$ by 125375.
Ans. $6984\frac{44661874}{879051867}$.
14. Divide $625\frac{166875}{934876}$ by 375875. *Ans.* $\frac{4675633}{2811172132}$.
15. Divide $\frac{625875}{934625}$ by 892756.
16. Divide $\frac{1378465375625875}{8763756259342534}$ by 1155.

IX.

TO MULTIPLY ONE FRACTION BY ANOTHER.

Multiply the numerators together for a new numerator, and the denominators for a new denominator.

1. Multiply $\frac{3}{4}$ by $\frac{1}{2}$.

OPERATION.

$$\tfrac{3}{4}\times\tfrac{1}{2}=\tfrac{3}{8},\ \textit{Ans.}$$

This process may be explained by referring to the definition of multiplication. Multiplying by a whole number consists in taking the multiplicand as many times as there are units in the multiplier. Multiplying by a fraction is taking a certain part of the multiplicand as many times as there are like parts of a unit in the multiplier. Multiplying by $\frac{1}{2}$, is taking $\frac{1}{2}$ of the multiplicand *once*. One half of $\frac{3}{4}$ is found by multiplying the denominator by 2. $\frac{3}{8}$ are $\frac{1}{2}$ of $\frac{3}{4}$. $\frac{3}{8}$ are taken *once*.

2. Multiply $\frac{8}{9}$ by $\frac{19}{30}$.

OPERATION.

$$\tfrac{8}{9}\times\tfrac{19}{30}=\tfrac{152}{270}=\tfrac{76}{135},\ \textit{Ans.}$$

Nineteen thirtieths of $\frac{8}{9}$ are taken once, in example 2; or one thirtieth of $\frac{8}{9}$ nineteen times. One thirtieth of $\frac{8}{9}$ is found by multiplying the denominator 9 by 30, and writing the product under 8, thus, $\frac{8}{270}$. Nineteen thirtieths are 19 times as much as one thirtieth; therefore by multiplying $\frac{8}{270}$ by 19, we have the true answer, $\frac{152}{270}=\frac{76}{135}$.

3. Multiply $\frac{17}{27}$ by $\frac{8}{11}$. *Ans.* $\frac{136}{297}$.
4. Multiply $\frac{13}{23}$ by $\frac{3}{5}$. *Ans.* $\frac{39}{115}$.
5. Multiply $\frac{18}{29}$ by $\frac{6}{7}$. *Ans.* $\frac{108}{203}$.
6. Multiply $8\frac{1}{2}$ by $6\frac{1}{3}$. *Ans.* $53\frac{5}{6}$.
7. Multiply $8\frac{7}{41}$ by $1155\frac{98}{103}$. *Ans.* $9444\frac{2418}{4223}$.

X.

TO DIVIDE ONE FRACTION BY ANOTHER.

Invert the divisor, and proceed as in multiplication.

1. Divide $\frac{4}{9}$ by $\frac{2}{3}$.

OPERATION.

$$\frac{3}{2}\times\frac{4}{9}=\frac{12}{18}=\frac{2}{3}, \textit{ Ans.}$$

This process deserves the careful attention of the pupil. To make the illustration of the rule more intelligible, let us suppose the above sum to be given thus: If a man pays $\frac{4}{9}$ of a dollar for $\frac{2}{3}$ of a yard of cloth, what will a yard cost?

If $\frac{2}{3}$ of a yard cost $\frac{4}{9}$ of a dollar, one third will cost one half of $\frac{4}{9}$, or $\frac{4}{18}$. If $\frac{1}{3}$ of a yard cost $\frac{4}{18}$, one yard will cost three times $\frac{4}{18}$, or $\frac{12}{18}=\frac{2}{3}$.

2. Divide $\frac{6}{11}$ by $\frac{8}{9}$. *Ans.* $\frac{27}{44}$.
3. Divide $\frac{8}{13}$ by $\frac{41}{53}$. *Ans.* $\frac{424}{533}$.
4. Divide $\frac{11}{15}$ by $\frac{54}{81}$. *Ans.* $1\frac{1}{10}$.
5. Divide $\frac{13}{25}$ by $\frac{611}{983}$.
6. Divide $17\frac{81}{93}$ by $64\frac{17}{21}$.
7. Divide $11543\frac{11873}{32334}$ by $334422\frac{113322}{995511}$.
8. Divide $1\frac{144225533}{663377448}$ by $2\frac{3366445577}{8992556699}$.
9. Divide $3\frac{33227788}{99665544}$ by $1\frac{9999}{82344}$.
10. Divide $61\frac{3}{4}$ by $81\frac{9}{11}$.
11. Divide $\frac{1}{2}$ by $\frac{3}{4}$.
12. Divide $\frac{1}{2}$ by $\frac{1}{2}$.
13. Divide $3\frac{1}{4}$ by $\frac{1}{6}$.

XI.

TO REDUCE FRACTIONS TO THE SAME DENOMINATOR.

Multiply the numerator and denominator of each fraction by the denominators of all the other fractions.

1. Reduce $\frac{3}{4}$, $\frac{7}{8}$, $\frac{5}{6}$ and $\frac{9}{11}$ to the same denominator.

OPERATION.

$$\tfrac{3}{4}\times8\times6\times11=\tfrac{1584}{2112}=\tfrac{3}{4}$$
$$\tfrac{7}{8}\times4\times6\times11=\tfrac{1848}{2112}=\tfrac{7}{8}$$
$$\tfrac{5}{6}\times4\times8\times11=\tfrac{1760}{2112}=\tfrac{5}{6}$$
$$\tfrac{9}{11}\times4\times8\times6=\tfrac{1728}{2112}=\tfrac{9}{11}$$

Ans. $\frac{1584}{2112}$, $\frac{1848}{2112}$, $\frac{1760}{2112}$, $\frac{1728}{2112}$.

It will be seen by inspecting the above process, that each fraction retains the same value after it is reduced, as it had before. And this is evident, because both terms of each fraction are multiplied by the same numbers.

2. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{8}$ to the same denominator.

Ans. $\frac{4}{8}$, $\frac{6}{8}$, and $\frac{1}{8}$.

3. Reduce $\frac{7}{8}$ and $\frac{5}{6}$ to the same denominator.

Ans. $\frac{42}{48}$ and $\frac{40}{48}$.

XII.

TO REDUCE A FRACTION OF A HIGHER DENOMINATION TO THAT OF A LOWER.

Multiply the given fraction by that number of the next lower denomination which makes a unit of the given denomination. Proceed in this manner until the fraction be reduced to the required denomination.

1. Reduce $\frac{1}{180}$ of a shilling to the fraction of a penny.

OPERATION.

$$\tfrac{1}{180}\times12=\tfrac{12}{180}=\tfrac{1}{15},\ \textit{Ans.}$$

As the value of shillings is 12 times greater than that of pence, therefore to reduce a fraction of a shilling to the fraction of a penny, the *number* of parts taken in the fraction of a shilling must be increased 12 times, or the *value* of the parts must be increased 12 times. Multiplying the numerator of a fraction multiplies the *number of parts* in a given fraction; dividing the denominator of a fraction increases the value of the number of parts in the given fraction. Hence, either of these ways may be adopted, according to the character of the sum.

2. Reduce $\frac{1}{280}$ of a pound to the fraction of a penny.

OPERATION.

$\frac{1}{280}\times 20=\frac{1}{14}\times 12=\frac{12}{14}=\frac{6}{7}$, *Ans.*

3. Reduce $\frac{1}{1200}$ of a pound to the fraction of a farthing.

$\frac{1}{1200}\times 20=\frac{1}{60}\times 12=\frac{1}{5}\times 4=\frac{4}{5}$, *Ans.*

4. Reduce $\frac{1}{9600}$ of a pound troy to the fraction of a grain.

$\frac{1}{9600}\times 12=\frac{1}{800}\times 20=\frac{1}{40}\times 24=\frac{24}{40}=\frac{3}{5}$, *Ans.*

5. Reduce $\frac{1}{225}$ of an ell English to the fraction of an inch.

$\frac{1}{225}\times 5=\frac{1}{45}\times 4=\frac{4}{45}\times 2\frac{1}{4}=\frac{36}{180}=\frac{1}{5}$, *Ans.*

6. Reduce $\frac{1}{110880}$ of a mile to the fraction of an inch.

$\frac{1}{110880}\times 8=\frac{1}{13860}\times 40=\frac{40}{13860}\times 16\frac{1}{2}=\frac{1320}{27720}=\frac{4}{7}$, *Ans.*

7. Reduce $\frac{3}{320}$ of a bushel to the fraction of a pint.

$\frac{3}{320}\times 4=\frac{3}{80}\times 8=\frac{3}{10}\times 2=\frac{3}{5}$, *Ans.*

XIII.

TO REDUCE A FRACTION OF A LOWER DENOMINATION TO THAT OF A HIGHER.

Divide the given fraction by that number of the same denomination which makes a unit of the next higher denomination. Proceed in this manner until the fraction be reduced to that of the required denomination.

1. Reduce $\frac{6}{7}$ of a penny to the fraction of a pound.

$\frac{6}{7}\div 12=\frac{6}{84}\div 20=\frac{6}{1680}=\frac{1}{280}$, *Ans.*

As 12 in the denomination of pence are equal to only 1 in the denomination of shillings, therefore the *number of parts* in the fraction of a penny must be diminished 12 times or the *value of the parts* must be diminished 12 times, to express, in the denomination of shillings, the value of the fraction of a penny. Either mode of reduction may be adopted according to the nature of the sum.

The same principle applies in every step of the process, in reducing a fraction of a lower denomination to that of a higher.

2. Reduce $\frac{3}{5}$ of a grain to the fraction of a pound.

Ans. $\frac{1}{9600}$

3. Reduce $\frac{1}{5}$ of an inch to the fraction of an ell English. *Ans.* $\frac{1}{225}$.

4. Reduce $\frac{4}{7}$ of an inch to the fraction of a mile. *Ans.* $\frac{1}{110880}$.

5. Reduce $\frac{3}{5}$ of a pint to the fraction of a bushel. *Ans.* $\frac{3}{320}$.

6. Reduce $\frac{4}{5}$ of a farthing to the fraction of a pound. *Ans.* $\frac{1}{1200}$.

XIV.

TO FIND THE LEAST COMMON MULTIPLE OF TWO OR MORE NUMBERS.

Divide by some number that will divide two or more of the given numbers without a remainder, and place the several quotients and the undivided numbers under the given numbers, and so continue to divide, until no number, exceeding unity, will divide two or more of them without a remainder. Multiply together all the divisors, quotients, and undivided numbers, and the product will be the least common multiple.

1. What is the common multiple of 3, 5, 8, and 10?

$$\begin{array}{r|rrrr} 5 & 3 & 5 & 8 & 10 \\ \hline 2 & 3 & 1 & 8 & 2 \\ \hline & 3 & 1 & 4 & 1 \end{array}$$

$10\times3\times1\times4\times1=120$, *Ans.*

To discover the propriety of this process, a close inspection of the example given, is necessary.

The first divisor (5) multiplied by the second divisor (2) is equal to 10, which are a multiple of 5. As 3 are not divided by 2 and 5, they remain 3, and enter into the process as a factor, $10\times3=30$, which are a multiple of 3, 5, and 10. But 30 are not a multiple of 8. Let 8 enter into the process as a factor, and the product then obtained will be a multiple of 8. Multiply 5 by 2 and 4, and we have 8 as a factor, and the product 40 is a multiple of 8. Then if 30 are a multiple of 3, 5, and 10, any number of times 30 are a multiple of the same numbers. And if 40 are a multiple of 8, so any number of times 40 are a multiple of 8. Therefore $40\times3=120$, a multiple of 3, 5, 8, and 10.

Again. The product of 3×5 is a multiple of 3 and 5; that product multiplied by 8 is a multiple of 3, 5, 8 and 10, because all these numbers are component parts of that product.

$3\times5=15$, a multiple of 3 and 5.
$3\times5\times8=120$, a multiple of 3, 5, 8, and 10.

2. What is the least common multiple of 3, 4, 8, and 12? *Ans.* 24.

3. What is the least common multiple of 4 and 6? *Ans.* 12.

4. How large must that vessel be, that may be filled by each one of the following measures, viz. 4 quarts, 6 quarts, 10 quarts, 12 quarts? *Ans.* 60 quarts.

XV.

TO FIND THE VALUE OF A FRACTION IN THE KNOWN PARTS OF A WHOLE NUMBER.

Multiply the given fraction by that number of the lower denomination which makes a unit of the denomination of the given fraction. If the product be an improper fraction, reduce it to a whole or mixed number. If there be a fraction over, proceed with it as before.

1. What is the value of $\frac{1}{12}$ of a £1?

$\frac{1}{12}\times20=\frac{20}{12}=1$, and $\frac{8}{12}$ or $\frac{2}{3}$ of a shilling.
$\frac{2}{3}\times12=\frac{24}{3}=8$ pence. *Ans.* 1 s. and 8 d.

For the demonstration of this rule, see sect. XII. Vulgar Fractions.

2. What is the value of $\frac{7}{8}$ of a guinea? *Ans.* 21 s. 9 d. $1\frac{1}{3}$ qr.

3. What is the value of $\frac{8}{9}$ of a pound troy? *Ans.* 10 oz. 13 pwts. 8 grs.

4. What is the value of $\frac{9}{17}$ of a rod? *Ans.* 144 ft. $19\frac{1}{17}$ in.

XVI.

TO REDUCE A MIXED QUANTITY OF WEIGHTS, MEASURES, ETC., TO THE FRACTION OF A WHOLE NUMBER.

Reduce the given numbers to the lowest denomination mentioned, for a numerator; and then reduce the whole number to the same denomination for the denominator of the required fraction.

1. Reduce 3 d. and 2 qrs. to the fraction of a pound.

OPERATION.

$$\begin{array}{r} 3-2 \\ 4 \\ \hline 14 \text{ numerator.} \end{array}$$

$1 \times 20 \times 12 \times 4 = 960$ denominator, $= \frac{7}{480}$, *Ans.*

One farthing is $\frac{1}{960}$ of a pound, and 14 farthings are fourteen times $\frac{1}{960}$ of a pound.

2. Reduce 2 gr. 15 lb. 4 oz. $5\frac{9}{11}$ dr. to the fraction of a cwt. *Ans.* $\frac{7}{11}$.

3. Reduce 7 oz. $1\frac{7}{9}$ dr. to the fraction of a pound. *Ans.* $\frac{4}{9}$.

4. What part of an ell English are 2 qr. 3 na. $0\frac{1}{4}$ in.? *Ans.* $\frac{5}{9}$.

5. What part of a mile are 6 fur. 30 rd. 12 ft. 8 in. $0\frac{12}{13}$ br.? *Ans.* $\frac{11}{13}$.

6. What part of a hhd. of beer are 42 gal.? *Ans.* $\frac{7}{9}$.

XVII.

TO ADD FRACTIONS.

When the denominators are the same, add the numerators; if the denominators are not the same, reduce the fractions to the same denominators.

1. Add $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$. *Ans.* $\frac{6}{5} = 1\frac{1}{5}$.

2. Add $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{4}{5}$.

$$\frac{1}{2}\times3=\frac{3}{6}\times5=\frac{15}{30}$$
$$\frac{1}{3}\times2=\frac{2}{6}\times5=\frac{10}{30}$$
$$\frac{4}{5}\times2=\frac{8}{10}\times3=\frac{24}{30}$$
$$\frac{15}{30}+\frac{10}{30}+\frac{24}{30}=\frac{49}{30}=1\frac{19}{30}, \textit{ Ans.}$$

3. Add $\frac{11}{12}$, $\frac{8}{10}$, $\frac{6}{13}$. *Ans.* $2\frac{278}{1560}$.

4. Add $\frac{1}{4}$, $\frac{1}{3}$, $\frac{4}{5}$, and $\frac{5}{6}$. *Ans.* $2\frac{78}{360}$.

XVIII.

TO SUBTRACT FRACTIONS.

Subtract the smaller numerator from the larger, when the denominators are the same; if the denominators are not the same, reduce the fractions to the same denominators.

1. From $\frac{7}{8}$ subtract $\frac{5}{8}$. *Ans.* $\frac{2}{8}$.
2. From $\frac{3}{4}$ take $\frac{1}{2}$. $\frac{1}{2}\times2=\frac{2}{4}$. $\frac{3}{4}-\frac{2}{4}=\frac{1}{4}$, *Ans.*
3. From $\frac{11}{13}$ take $\frac{2}{3}$. $\frac{33}{39}-\frac{26}{39}=\frac{7}{39}$, *Ans.*
4. From $\frac{10}{11}$ take $\frac{4}{5}$. *Ans.* $\frac{6}{55}$.

EXAMPLES FOR PRACTICE

IN THE PRECEDING RULES OF VULGAR FRACTIONS.

1. Reduce $\dfrac{3\frac{1}{2}}{4\frac{1}{2}}$ of $\dfrac{5\frac{1}{3}}{8\frac{2}{3}}$ of $\dfrac{6\frac{1}{7}}{8\frac{1}{9}}$ to a simple fraction.

2. Reduce $\frac{11}{13}$, $\frac{3}{4}$ of $1\frac{1}{4}$, $\frac{9}{11}$, and $\frac{5}{7}$ to the same denominator. *Ans.* $\frac{13552}{16016}$, $\frac{15015}{16016}$, $\frac{13104}{16016}$, $\frac{11440}{16016}$.

3. Reduce $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{8}$ to a simple fraction. *Ans.* $\frac{1}{4}$.

4. What is the value of $376\frac{11}{18}$ acres of land, at \$$75\frac{3}{8}$ per acre? *Ans.* \$28387,$06\frac{1}{4}$.

5. What is the cost of $8\frac{1}{2}$ cords of wood, at \$$2\frac{3}{8}$ per cord? *Ans.* \$$20\frac{3}{16}$, or \$20,$18\frac{3}{4}$, or \$20,1875.

6. What is the value of $9\frac{1}{10}$ pounds of wool, at $33\frac{1}{3}$ cents per pound?

OPERATION.

$3|91$

Ans. $3,03⅓

Or thus: 9,1×,33⅓=$3,0333⅓, *Ans.*

7. What is the value of 18¾ pounds of butter, at 16⅔ cents per pound? *Ans.* $3⅛, or $3,125.

This sum may be wrought thus:

$6|18,75$

$3,125, *Ans.*

8. What is the cost of 2755 pounds of butter, at 12½ cents per pound?

OPERATION.

$8|2755$

$344⅜, or $344,375, *Ans.*

9. What is the cost of ten doz. of hats, at 2¼ dollars per hat? *Ans.* $270.

10. What will 40 skeins of yarn cost, at 66⅔ cents per skein? *Ans.* $26⅔.

11. What will 80 skeins of yarn cost, at 33⅓ cents per skein? *Ans.* $26⅔.

12. What will 160 skeins of yarn cost, at 16⅔ cents per skein? *Ans.* $26⅔.

13. What will 320 skeins of yarn cost, at 8⅓ cents per skein? *Ans.* $26⅔.

14. What will 640 skeins of yarn cost, at 4⅙ cents per skein? *Ans.* $26⅔.

15. What will 426⅔ skeins of yarn cost, at 6¼ cents per skein? *Ans.* $26⅔.

VIII. Reduction.

Reduction is changing one denomination to another without altering its value. Thus, 1 pound is 20 s.; 20 s. are 240 d.; and 240 d. are 960 qrs. It is plain, therefore, that £1 is equal to 960 qrs. Changing pounds to shillings, shillings

to pence, and pence to farthings, is called *Reduction.* On the other hand, changing farthings to pence, pence to shillings, and shillings to pounds, is called Reduction. Hence, Reduction is of two kinds, Reduction *Ascending* and *Descending.* Reduction ascending is reducing a lower denomination to a higher. Reduction descending is reducing a higher denomination to a lower.

RULE.

I. TO REDUCE A LOWER DENOMINATION TO A HIGHER.

Divide the given sum by that number which makes a unit of the next higher denomination. Proceed in this manner until the sum is reduced to the required denomination.

II. TO REDUCE A HIGHER DENOMINATION TO A LOWER.

Multiply the given sum by that number of the next lower denomination, which makes a unit of the given denomination Proceed in this manner, until the sum is reduced to the denomination required.

1. Reduce 1784 pence to pounds.

```
1784|12
    | 148-8|20
           |£7 - 8s. - 8 d. Ans.
```

In reducing pounds to farthings, the following things will be noticed. Shillings are 20th parts of a pound, pence are 240th parts of a pound, or 12th parts of a shilling, and farthings are 960th parts of a pound, 48th parts of a shilling, or 4th parts of a penny. This may be seen in the following example.

2. Reduce 20 pounds, 6 s. 7 d. 3 qrs. to farthings.

```
£20  6 s.  7 d.  3 qrs.
  20
 ----
 406 s. or 20th parts of a pound.
  12
 ----
4879 d. or 240th parts of a pound, and 12th parts
   4                                 [of a shilling.
-----
19519 qrs. or 960th parts of a pound, 48th parts of
                 [a shilling, and 4th parts of a penny.
```

NOTE.—The intelligent pupil will perceive that the shillings, pence, and farthings in the given sum, are added to the several products of their own denomination; and this rule is to be observed in all similar examples in reduction.

Reducing pounds to farthings, is reducing them to 960th parts, or to a denomination in which a unit is $\frac{1}{960}$ of a pound. Hence, reducing farthings to pounds is finding how many 960th parts there are in the given number of 960th parts, or farthings, when expressed in pounds, or units taken in the denomination of pounds.

3. Reduce £21 18 s. 11 d. 3 qrs. to farthings, or 960th parts of a pound.

```
        £21  18 s.  11 d.  3 qrs.
          20
        ----
         438 s. or 20th parts of a £.
          12
        ----
        5267 d. or 240th parts of a £.
           4
        -----
Ans.   21071 qrs. or 960th parts of a £.
```

PROOF.

1 £=960 times $\frac{1}{960}$ part of a pound, or 960 qrs.
21 £=960×21=20160 qrs. or 960th parts of a pound.
1 s.=48 times $\frac{1}{960}$ part of a pound, or 48 qrs.
18 s.=48×18=864 times $\frac{1}{960}$ of a £, or 864 qrs.
1 d.=4 times $\frac{1}{960}$ part of a pound, or 4 qrs.
11 d.=4×11=44 qrs. or 44 times $\frac{1}{960}$ part of a pound.
1 qr.=$\frac{1}{960}$ part of a pound.
3 qrs.=$\frac{1}{960}$×3=3 times $\frac{1}{960}$ part of a pound, or 3 qrs.

```
£21=20160 qrs. or 960ths of a pound.
   18 s. =864 qrs. or   "   "   "
   11 d. = 44 qrs. or   "   "   "
   3 qrs.=  3 qrs. or   "   "   "
          -----
          21071 qrs. or  "   "   "
```

4. Reduce 938763 farthings to pounds, shillings, pence, and farthings. *Ans.* 977£, 17 s. 6 d. 3 qrs.

5. Reduce 1656 pence to pounds. Ans. 6 £, 18 s.

6. Reduce 20 £, 13 s. 4 d. 2 qrs. to farthings.
Ans. 19842.

7. In 7 £, 8 s. 8 d. how many farthings? *Ans.* 7136.

TABLES.

UNITED STATES MONEY.

10 mills make one cent, marked c.
10 cents " " dime, " d.
10 dimes " " dollar, " $.
10 dollars " " eagle. " E.

Mills.		Cents.		Dimes.		Dollars.		Eagle.
10	=	1						
100	=	10	=	1				
1000	=	100	=	10	=	1		
10000	=	1000	=	100	=	10	=	1

ENGLISH MONEY.

4 farthings make one penny, marked d.
12 pence " " shilling, " s.
20 shillings " " pound, " £.

qrs.		d.		s.		£.
4	=	1				
48	=	12	=	1		
960	=	240	=	20	=	1

TROY WEIGHT.

By this weight are weighed gold, silver, and jewels.

24 grains make one pennyweight, marked pwt.
20 pennyweights " ounce, " oz.
12 ounces " pound, " lb.

grs.		pwt.		oz.		lb.
24	=	1				
480	=	20	=	1		
5760	=	240	=	12	=	1

NOTE.—"The original of all weights, used in England, was a grain or corn of wheat, gathered out of the middle

of the ear; and being well dried, 32 of them were to make one pennyweight, 20 pennyweights one ounce, and 12 ounces one pound. But in later times, it was thought sufficient to divide the same pennyweight into 24 parts, still called grains, being the least weight now in common use; and from hence the rest are computed."*

1. Reduce 2 pounds, 8 oz. 19 pwt. 21 grs. to grains.

```
   lb. oz. pwt. grs.
    2   8   19   21            1 pound or unit.
   12                         12
   ----                      ----
   32 oz. or 12th parts.      12  12th parts.
   20                         20
   ----                      ----
  659 pwts. or 240th parts.  240  240th parts.
   24                         24
  -----                      ----
15837 grains, or 5760th parts 960
             [of a pound.     480
                             ----
                             5760  parts.
```

2. Reduce 9 pounds, 8 oz. to grains. *Ans.* 55680.

3. How many ounces in 246 ingots of silver, each weighing 4 lb. 6 oz.? *Ans.* 13284 ounces.

AVOIRDUPOIS WEIGHT.

By this weight are weighed butter, cheese, pork, beef, hay, wool, tallow, iron, steel, lead, zinc, and every thing of a coarse and drossy nature.

16 drams	make one	ounce,	marked oz.
16 ounces	" "	pound,	" lb.
25 pounds	" "	quarter,	" qr.
4 quarters	" "	hundred weight	" wt.
20 hundred wt.	"	ton,	" ton.

dr.		oz.								
16	=	1		lb.						
256	=	16	=	1		gr.				
6400	=	400	=	25	=	1		cwt.		
25600	=	1600	=	100	=	4	=	1		ton.
512000	=	32000	=	2000	=	80	=	20	=	1

* Greenleaf's Arithmetic.

NOTE.—This table is for net weight only. But in gross weight 112 pounds make 100, and 28 pounds one quarter. This weight is seldom used.

1. Reduce 1 ton, 17 cwt. 3 qr. 17 lbs. 13 oz. 5 dr. to drams. *Ans.* 970965 drams.

2. Reduce 970965 drams to tons, &c.
Ans. 1 t. 17 cwt. 3 qr. 17 lb. 13 oz. 5 dr.

APOTHECARIES' WEIGHT.

This weight is used for compounding medicines.

20 grains make one scruple, marked sc. or ℈
3 scruples " " dram, " dr. or ʒ
8 drams " " ounce, " oz. or ℥
12 ounces " " pound, " lb. or ℔

gr.		sc.						
20	=	1		dr.				
60	=	3	=	1		oz.		
480	=	24	=	8	=	1		lb.
5760	=	288	=	96	=	12		1

1. How many grains in 48 lbs. 6 oz.? *Ans.* 279360.
2. In 27960 grains how many pounds? *Ans.* 48 lbs. 1 oz.
3. In 216000 grains how many pounds.?
Ans. 37 lbs. 6 oz.
4. In 12 lbs. 1 oz. 2 dr. 0 sc. 1 gr. how many grains?
Ans. 69721.
5. In 69721 grs. how many pounds, oz. &c.
Ans. 12 lbs. 1 oz. 2 dr. 0 sc. 1 gr.

CLOTH MEASURE.

This measure is used in measuring cloth.

2¼ inches make one nail, marked na.
2 nails " " eighth, " e.
2 eighths " " quarter, " qr.
4 quarters " " yard, " yd.

inches		nail						
2¼	=	1		eighth				
4½	=	2	=	1		quarter.		
9	=	4	=	2	=	1		yard
36	=	16	=	8	=	4	=	1

Other measures which are sometimes used.

3 quarters	make one ell	Flemish,	marked	E. F.
5 "	" "	" English,	"	E. E.
6 "	" "	" French,	"	E. Fr.
37⅕ inches	" "	" Scotch,	"	E. S.

1. In 9 yds. 1 qr. 1 e. 1 n. how many inches?
Ans. 339,75.

2. In 339,75 inches how many yards, quarters, eighths, &c.? *Ans.* 9 yds. 1 qr. 1 e. 1 n.

LONG MEASURE

Long measure is used in measuring distances, where breadth or thickness is not considered, but length only.

3 barley-corns	make one	inch,	marked	in.
12 inches	" "	foot,	"	ft.
3 feet	" "	yard,	"	yd.
5½ yds, or 16½ ft.	" "	rod or pole,		rd.
40 rods	" "	furlong,	"	fur.
8 furlongs	" "	mile,	"	m.
3 miles	" "	league,	"	lea.
69½ miles nearly	" "	degree,	"	deg. or °
360 degrees	" "	circle of the		earth.

bar.	in.								
3=	1	ft.							
36=	12=	1=	yds.						
108=	36=	3=	1=	rd.					
954=	198=	16½=	5½=	1	fur.				
23760=	7920=	660=	220=	40=	1	mi.			
190080=	63360=	5280=	1760=	320=	8=	1	lea.		
570240=	190080=	15840=	5280=	960=	24=	3=	1	de.	
13210560=	4403520=	366960=	122320=	22240=	556=	69½=	23⅙=	1	c
4755801600=	1585267200=	132105600=	44035200=	8006400=	200160=	25020=	8340=	360=	1

inches.	links.				
7,92=	1	poles.			
198, =	25=	1	chains.		
792, =	100=	4=	1	furlongs.	
7920, =	1000=	40=	10=	1	mile.
63360, =	8000=	320=	80=	8=	1

1. In 360° 25 m. how many barley-corns?
Ans. 4760553600.

2. In 4760553600 barley-corns, how many degrees and miles? *Ans.* 360° 25 m.

3. How many barley-corns would reach from Albany to New York, the distance being 160 miles? *Ans.* 30412800.

SQUARE MEASURE.

In this measure length and breadth are considered.

144 square inches	make	one	square foot,	marked ft.
9 square feet	"	"	square yard,	" yd.
30¼ square yards	"	"	square rod or pole,	" p.
272¼ square feet	"	"	"	p.
40 square rods	"	"	rood,	" r.
4 roods	"	"	acre,	" a.
640 acres	"	"	square mile,	" s. m.

in.							
1	prines.						
12=	1	ft.					
144=	12=	1	yd.				
1296=	108=	9=	1	p.			
39204=	3267=	272¼=	30¼=	1	r.		
1568160=	130680=	10890=	1210=	40=	1	a.	
6272640=	522720=	43560=	4840=	160=	4=	1	m.
4014489600=	334540300=	27878400=	3097600=	102400=	2560=	640=	1

1. In 1 m. 2 a. 3 r. 5 p. 1 yd. 7 ft. 11 pr. 5 in. how many inches? *Ans.* 4031937821.

2. In 4031937821 inches, how many miles, acres, &c.? *Ans.* 1 m. 2 a. 3 r. 5 p. 1 yd. 7 ft. 11 pr. 5 in.

SOLID OR CUBIC MEASURE.

In this measure length, breadth, and thickness are considered.

1728 inches	make	one	foot.
27 feet	"	"	yard.
40 feet of hewn timber, or 50 feet of round timber,	"	"	ton.
128 feet	"	"	cord of wood.
16 feet	"	"	cord foot.
8 cord feet	"	"	cord of wood.

in.					
1	pr.				
12=	1	ft. board measure.			
144=	12=	1	solid ft.		
1728=	144=	12=	1	cord ft.	
27648=	2304=	192=	16=	1	cord.
221184=	18432=	1538=	128=	8=	1

1. In 5 cords, 7 cord ft. 95 solid ft. 4 ft. board measure, 7 primes, 11 inches, how many inches? *Ans.* 1464287.

2. In 1464287 inches, how many cords, cord ft. &c.?
Ans. 6 cords, 4 cord ft. 15 solid ft. 4 ft. board [measure, 7 primes, 11 inches.

TIME.

60 seconds	make one	minute,	marked	m.
60 minutes	" "	hour,	"	h.
24 hours	" "	day,	"	d.
7 days	" "	week,	"	w.
4 weeks	" "	month,	"	mo.
13 months, 1 day, 6 hours, or 365 days, 6 hours,	}	Julian year,	"	y.
12 calendar months make		one year,	"	y.

sec.	m.					
60=	1	h.				
3600=	60=	1	d.			
86400=	1440=	24=	1	w.		
604800=	10080=	168=	7=	1	mo.	
2419200=	40320=	672=	28=	4 =	1	y.
31557600=	525960=	8766=	$365\frac{1}{4}$=	$52\frac{5}{28}$=	$13\frac{5}{112}$=	1

1. In 3 y. 5 mo. 3 w. 2 d. 9 h. 12 m. 40 sec. how many seconds? *Ans.* 108789160.

2. In 108789160 seconds, how many years, months, days, &c.? *Ans.* 3 y. 5 mo. 3 w. 2 d. 9 h. 12 m. 40 sec.

CALENDAR MONTHS.

January,	1st	month, has	31	days.
February,	2d	" "	28	"
March,	3d	" "	31	"

April,	4th	month, has	30	days.
May,	5th	" "	31	"
June,	6th	" "	30	"
July,	7th	" "	31	"
August,	8th	" "	31	"
September,	9th	" "	30	"
October,	10th	" "	31	"
November,	11th	" "	30	"
December,	12th	" "	31	"

30 days has September,
April, June, and November;
All the rest have 31,
But February alone;
Of this short month the common rate
Of days, is only 28;
But when leap year has more time,
Its number then is 29.

CIRCULAR MEASURE.

This measure is used in reckoning latitude and longitude; also in computing the revolutions of the planets.

60 seconds (″)	make one minute,	marked	′.
60 minutes	" " degree,	"	°.
30 degrees	" " sign,	"	s.
12 signs, 360 degrees,	" the circle of the zodiac.		

$$
\begin{array}{r}
1'' \\
60 = \quad 1' \\
3600 = \quad 60 = \quad 1^\circ \\
108000 = 1800 = 30 = 1\text{ s.} \\
1296000 = 21600 = 360 = 12 = 1\text{ cir.}
\end{array}
$$

1. In 1874407 sec. how many circles, &c.?
Ans. 1 cir. 5 s. 10° 40′ 7″.

2. In 1 circle, 5 s. 10° 40′ 7″ how many seconds?
Ans. 1874407.

WINE MEASURE.

This measure is used in measuring wines, and nearly all kinds of liquors.

4 gills	make one pint,	marked pt.
2 pints	" " quart,	" qt.

4 quarts	make one	gallon,	marked	gal.
42 gallons	"	" tierce,	"	tier.
63 gallons	"	" hogshead,	"	hhd.
2 tierces	"	" puncheon,	"	pun.
2 hogsheads	"	" pipe or butt,	"	pi.
2 pipes or 4 hhd.	"	" tun,	"	tun.

inches,	pints.							
$28\frac{7}{8}$=	1	quarts.						
$57\frac{3}{4}$=	2=	1	gallons.					
231 =	8=	4=	1	tierces.				
9702 =	336=	168=	42=	1	hogsheads.			
14553 =	504=	252=	63=	1,5=	1	puncheons.		
19404 =	672=	336=	84=	2	=$1\frac{1}{3}$=	1	pipes.	
29106 =	1008=	504=	126=	3	=2	=1,5=	1	tun.
58212 =	2016=	1008=	252=	6	=4	=3	=2=	1

1. In 1 tun, 1 pi. 1 pun. 1 hhd. 1 tier. 20 gal. 3 qt. 1 p. how many inches? *Ans.* 135799,125.

2. In 1357999,125 inches, how many tuns, &c.?
Ans. 1 tun, 1 pi. 1 pun. 1 hhd. 1 tier. 20 gal. 3 qt. 1 pt.

ALE OR BEER MEASURE.

This measure is used for measuring milk, ale, and beer.

2 pints	make one	quart,	marked	qt.
4 quarts	"	" gallon,	"	gal.
36 gallons	"	" barrel,	"	bar.
54 gallons	"	" hogshead,	"	hhd.
2 hogsheads	"	" butt,	"	butt.
2 butts	"	" tun,	"	tun.

inches.	pints.								
35,25=	1	qt.							
70,5 =	2=	1	gal.						
282 =	8=	4=	1	fir.					
2538 =	72=	36=	9=	1	kil.				
5076 =	144=	72=	18=	2=	1	bar.			
10152 =	288=	144=	36=	4=	2=	1	hhd.		
15228 =	432=	216=	54=	6=	3=	1,5=	1	pun.	
20304 =	576=	288=	72=	8=	4=	2	=$1\frac{1}{3}$=	1	butt.
30456 =	864=	432=	108=	12=	6=	3	=2	=1,5=	1

In Massachusetts, the barrel for cider and beer is 32 gal. The ale gallon contains 282 cubic solid inches.

1. In 3 butts, 2 pun. 5 hhd. 3 bar. 6 kilderkins, 9 firkins, 7 gal. 5 qts. 3 pts. how many cubic inches? *Ans.* 294302¼.

2. In 3807000 cubic inches how many butts? *Ans.* 125.

DRY MEASURE.

This measure is used in measuring all kinds of grain, fruit, roots, coal, &c.

2 pints (pts.)	make one	quart,	marked	qts.
2 quarts	" "	pottle,	"	pot.
2 pottles	" "	gallon,	"	gal.
2 gallons	" "	peck,	"	pk.
4 pecks	" "	bushel,	"	bu.
2 bushels	" "	strike,	"	str.
2 strikes	" "	coom,	"	co.
2 cooms	" "	quarter,	"	qr.
4 quarters	" "	chaldron,	"	ch.
4½ quarters	" "	ch. in London	"	ch.
5 quarters	" "	wey,	"	wey,
2 weys	" "	last,	"	last.

inches.								
33,6=	1 pint.							
67,2=	1 qt.	gal.						
268,8=		1	peck.					
537,6=		2=	1	bushel.				
2150,4=		8=	4=	1	str.			
4300,4=		16=	8=	2=	1	coom.		
8601,6=		32=	16=	4=	2=	1	quarter	
17203,2=		64=	32=	8=	4=	2=	1	wey.
86016 =		320=	160=	40=	20=	10=	5=	1 last.
172032 =		640=	320=	80=	40=	20=	10=	2=1

1. In 5 lasts, 3 weys, 2 qrs. 1 co. 1 str. 1 bu. 3 pk. 1 gallon, 2 qt. 1 pt. how many cubic inches? *Ans.* 1169716,8.

2. In 1169716,8 inches, how many lasts, weys, qrs. &c.?
Ans. 5 lasts, 3 weys, 2 qrs. 1 co. 1 str. 1 bu. 3 pk. 1 gal. 2 qt. 1 pt.

The following denominations are frequently used.

12 particular things	make	one	dozen.
12 dozen	"	"	" gross.
12 gross, or 144 dozen		"	great gross.
20 particular things	"	"	score.
6 points	"	"	line.
12 lines	"	"	inch.
6 feet	"	"	fathom.
24 sheets	"	"	quire.
20 quires	"	"	ream.
4 inches	"	"	hand.
5 feet	"	"	geometrical pace.

MISCELLANEOUS EXAMPLES.

1. How many guineas, of 28 shillings each, are there in £84? *Ans.* 60.

2. In 75 pistoles, of 22 s. each, how many pounds? *Ans.* £82 10 s.

3. In 75 pistoles how many dollars? *Ans.* 275.

4. In 42 guineas how many dollars? *Ans.* 196.

5. In 50807 moidores, of 36 s. each, how many pieces of coin, each 4 s. 6 d.? *Ans.* 406456

6. In 59 lbs. 13 pwts. 5 grs. how many grains? *Ans.* 340157.

7. In 420 quarter guineas how many moidores? *Ans.* 81$\frac{2}{3}$.

8. In 274 marks, each 17 s. 9 d. and 87 nobles, each 8 s. 11d. how many pounds? *Ans.* £281 19 s. 3 d.

9. Suppose a young man to be 20 years old, how many seconds has he lived, allowing a year to be 365 days, 6 hours? *Ans.* 631152000.

10. How many times will a wheel, which is 18 feet 4 inches in circumference, turn over in passing through the space of 2 miles?

11. In 60 years how many seconds? *Ans.* 1893456000.

12. How many sparrows, at the rate of 10 for a penny, would pay for an ox worth £20? *Ans.* 48000.

13. In 100 dollars, 40 guineas, 64 moidores, and 3d. how many pounds sterling? *Ans.* £201 4 s. 6 d.

14. In 9 cords and 48 cubic feet how many cord feet? *Ans.* 75.

15. In 25 cords how many feet, board measure? *Ans.* 38400.

16. In 8 tuns of wine how many pints? *Ans.* 16128.

17. In £29 how many groats, at 4 d. each? *Ans.* 1740.

18. In 473 French crowns, at 6 s. 8 d. each, how many three-penny pieces? *Ans.* 12613⅓.

19. In 473 half Johannes, at 48 s. each, how many 1½ penny pieces? *Ans.* 18816.

20. In 1259 groats how many farthings, pence, shillings, and guineas? *Ans.* 20144 qrs. 5036 d. 419 s. 8 d. 14 guineas, 27 s. 8 d.

21. What will 87 bushels and ½ a peck of meal come to, at 2 cents per quart? *Ans.* 50 dollars.

IX. Compound Numbers.

Numbers of the same name, or kind, are called *simple numbers.* Numbers of different denominations are *compound.*

COMPOUND ADDITION.

Compound Addition is putting together numbers of different denominations, to find the whole sum, or amount.

RULE.—*Place numbers of the same denomination directly under each other. Begin at the right hand, and add successively the figures in each column as in simple numbers, observing to carry for that number of the given column,*

which makes a unit of the next higher denomination, or the next left-hand column.

1. Add £2 8 s. 11 d., £3 9 s. 10 d., £8 19 s. 4 d. 3 qr., £25 11 s. 9d. 2 qr.

OPERATION.

20		12	4
2	8	11	
3	9	10	
8	19	4	3
25	11	9	2

Ans. £40 9 s. 11d. 1 qr.

In this example, I place pounds under pounds, shillings under shillings, pence under pence, farthings under farthings. I begin at the right hand, and add the column of farthings. As 4 farthings make a penny, I place 4 over farthings, and divide by it, setting down the remainder under farthings, and carrying the quotient to the next left-hand column. I proceed in this manner till I come to the last column. Pounds being the highest denomination in English money, I add them, and set down the whole amount. Every 4 farthings make 1 penny; therefore, as many times as 4 are contained in the given number of farthings, so many pence there would be. If there be a remainder, in dividing farthings by farthings, it will be farthings, and must be written down according to the rule, that is, under the column of farthings. As 12 pence make 1 shilling, as many times as 12 are contained in the given number of pence, so many shillings there will be. If there be a remainder in dividing pence by 12, it will be pence, and must be written under the column of pence, according to the rule. The same principle applies to all the operations of carrying in compound addition.

2. A man sold a horse for £20 8 s. 12 d., a chaise for £60 9 s. 9 d., a yoke of oxen for £25 11 s. 8 d.; what amount of money did he receive for all he sold?

Ans. £106 10 s. 5 d.

3. What is the amount of the following sums?

£	s.	d.
28	6	$9\frac{1}{2}$
15	16	$11\frac{1}{4}$
31	13	$10\frac{1}{2}$
14	16	9
17	17	$7\frac{1}{4}$
32	18	$8\frac{1}{4}$
Ans. 141	10	$7\frac{3}{4}$

4. Add

yd.	qr.	na.	in.
37	3	3	2
61	3	1	1
13	2	2	2
32	1	1	1
61	2	2	2
22	1	3	0
Am't. 229	3	3	$1\frac{1}{4}$

5. Add

	deg.	m.	fur.	rd.	ft.	in.	br.
	17	69	7	39	16	11	2
	61	62	3	17	12	9	1
	16	16	6	16	13	10	2
	48	19	3	15	15	6	1
	17	58	6	33	14	7	1
	33	35	5	19	9	9	2
Amount.	195	54	5	24	1	1	0

6. Add

	S.	°	′	″
	5	39	54	39
	4	15	36	25
	10	15	46	48
	9	18	26	31
	2	24	31	22
Amount.	29	34	15	45

7.

	y.	mo.	w.	d.	h.	m.	s.
	54	10	3	5	22	27	54
	31	8	2	3	15	18	34
	42	7	3	4	16	10	25
	31	9	1	3	10	9	18
Ans.	160	$9\frac{1}{56}$	3	3	16	6	11

COMPOUND SUBTRACTION.

Compound Subtraction is taking a sum of different denominations, and of less value, from a sum of different denominations, and of greater value.

RULE.—*Place the numbers as in compound addition, and, commencing at the right hand, subtract the figure in the right-hand column from the figure above it in the minuend, and place the remainder directly under the right-hand column. If the figure in the minuend be less than the figure under it, in the subtrahend, add to it the figure above it, and from that amount subtract the figure in the subtrahend, and place the remainder directly below. Whenever the number above the minuend is added to it, 1 must be added to the next left-hand figure in the subtrahend before subtracting.*

PROOF.—*Add the remainder to the subtrahend, and if the work is right, the amount will agree with the minuend.*

1. From £82 1 s. 1 d. 1 qr. take £81 19 s. 11 d. 3 qrs.

OPERATION.

		20	12	4
	82	1	1	1
	81	19	11	3
Ans.		1	1	2

2. A note bearing date Dec. 28, 1826, was paid Jan. 2, 1827; how long was it at interest?

OPERATION.

	12	30
1827	0	2
1826	11	28
0	0	4

In this example, I set down the later date, 1827, and 2 days on the next year. As there are only 2 days on the year after '27, I place a cipher in the column of months. Under this date I write the former one, 1826 years, 11 months, and 28 days, and subtract according to the rule.

3. What is the difference of time between the 13th day of June, 1838, and the 27th day of April, 1841?

Ans. 2 yrs. 10 mo. 14 days.

COMPOUND MULTIPLICATION.

Compound Multiplication teaches to repeat a compound number a certain number of times, or as many times as there are units and parts of units in the multiplier.

RULE.—*Place the multiplier under the right-hand column of the multiplicand, and multiply the figures of the lowest denomination by the multiplier, setting down as in addition. Multiply the figures of the next higher denomination by the multiplier, and to this product add the figure or figures, (if any,) that were to be carried to this denomination from the one lower. Proceed in this manner till you come to the highest denomination in the table employed, when you set down the whole number.*

1. Multiply £82 8 s. 11 d. 3 qrs. by 8.

OPERATION.

		20	12	4
	£82	8 s.	11 d.	3 qrs.
				8
Ans.	£659	11	10	0

2. Multiply £3 12 s. 9 d. by 9. *Ans.* £32 14 s. 9 d.

3. Multiply 18 lb. 15 oz. 13 drs. by 9.
Ans. 1 cwt. 2 qrs. 20 lb. 14 oz. 5 dr.

4. A man bought 4 lots of wheat, each of which contained 8 bushels, 3 pecks, and 2 quarts; how much wheat did he buy? *Ans.* 35 bushels and 1 peck.

COMPOUND DIVISION.

Compound Division teaches to divide a compound number.

RULE.—*Place the divisor at the right hand of the dividend, and seek how many times it is contained in the highest*

denomination of the dividend. Place the quotient figure under the divisor, and multiply as in simple numbers. Subtract, and reduce the remainder to the next lower denomination, and to the product add this lower denomination. Divide as before, and proceed in this manner to the close.

1. Divide £62 16 s. by 24.

OPERATION.

```
62 - 16 | 24
48          Ans. £2 12 s. 4 d.
--
14
 20
---
296
24
---
 56
 48
---
  8
 12
---
 96
 96
---
 00
```

2. Bought 139 yards of cloth for £461 11 s. 11 d.; what was the cost per yard? *Ans.* £3 6 s. 5 d.

3. Bought 84 pipes of brandy, containing 9468 gal. 1 qt. 1 pt.; how much in a pipe?

Ans. 112 gal. 2 qts. 1 pt. 3 gills.

X. Reduction of Currencies.

Previous to 1786, all calculations in money, throughout the United States, were made in pounds, shillings, pence and farthings. But these denominations, although the same *in name* as those of English money, had different values in different places. Thus, one dollar is reckoned

In	England,	4 s. 6 d. called English or sterling money.
"	Canada and Nova Scotia,	5 s. called Canada currency.
"	New England, Virginia, Kentucky, and Tennessee,	6 s. called New England currency.
"	New York, Ohio, and North Carolina,	8 s. called New York currency.
"	New Jersey, Pennsylvania, Delaware and Maryland,	7 s. 6 d. called Pennsylvania currency.
"	South Carolina, and Georgia,	4 s. 8 d. called Georgia currency.

TO REDUCE POUNDS, SHILLINGS, PENCE, AND FARTHINGS, OF ANY GIVEN CURRENCY, TO FEDERAL MONEY.

Reduce the pounds and shillings in the given sum to pence, and the farthings to the decimal of a penny. Then divide by the number of pence (in the currency to be reduced) that make a dollar, and the quotient will be the answer.

NOTE.—*The process of reduction may be abbreviated in certain cases, by reducing the given sum to shillings and decimal of a shilling, and dividing by the number of shillings (of the currency to be reduced) that make a dollar.*

1. Reduce £6 11 s. 6 d. 1 qr. to federal money.

4 s. 6 d.=1 dollar.
12
54=9×6.

6 - 11 - 6 - 1
20
131
12
6|1578,25
9|263,04$\frac{1}{6}$
Ans. $29,22$\frac{37}{54}$, or $29,2268$\frac{14}{27}$.

4|1,0
,25

2. Reduce £6 11 s. 6 d. 1 qr. Pennsylvania currency, to federal money.

7 s. 6 d.=1 dollar.
12
90=10×9.

10 | 1578,25
9 | 157,825
Ans. $17,536$\frac{1}{9}$.

3. Reduce £6 11 s. 6 d. 1 qr. Canada currency, to federal money. The pounds, shillings, pence and farthings, reduced to pence and the decimal of a penny, produce the same expression as before, namely, 1578,25.

5 s.=1 dollar.
12 s.

60=10×6.

Divide by 10, by removing the point one figure to the left, and then divide by 6.

6 | 157,825

Ans. $26,304$\frac{1}{6}$.

4. Reduce £6 11 s. 6 d. 1 qr. Georgia currency, to federal money.

4 s. 8 d.=1 dollar.
12

56=8×7.

8 | 1578,25

7 | 197,28125

Ans. $28,18303$\frac{4}{7}$.

5. Reduce £6 11 s. 6 d. 1 qr. New England currency, to federal money.

6 s.=1 dollar.
12

72=8×9.

Divide by 8 and 9.

8 | 1578,25

9 | 197,28125

Ans. $21,92013$\frac{8}{9}$.

6. Reduce £6 11 s. 6 d. 1 qr. New York currency, to federal money.

The whole number and decimal will be the same in each of the currencies.

8 s.=1 dollar.
12

96=8×12.

Divide by 8 and 12.

8 | 1578,25

12 | 197,28125

Ans. $16,44010416$\frac{2}{3}$.

TO REDUCE FEDERAL MONEY TO POUNDS, SHILLINGS, PENCE, AND FARTHINGS OF THE SEVERAL CURRENCIES.

Multiply the given sum by the number of shillings (of the required currency) that make a dollar, and divide by 20, the quotient will be pounds; the remainder will be shillings and decimal parts of a shilling.

1. Reduce $118,25 to pounds, shillings, pence, and farthings of New England currency.

$$\begin{array}{r} 118,25 \\ 6 \\ \hline 709,50 \end{array}$$

This product is in shillings, N. E. currency. Divide by 20 s. and the quotient will be pounds, according to the common rule of reduction.

$$20\,|\,70'9,50$$

Ans. £35 9s. 6d.

2. Reduce $118,25 to pounds, shillings, pence and farthings of Canada currency.

$$\begin{array}{r} 118,25 \\ 5 \\ \hline 20\,|\,59'1,25 \end{array}$$

Ans. £29 11s. 3 qrs.

3. Reduce $118,25 to the denominations of English money.

Ans. £26 12 s. $1\frac{1}{2}$ d.

4. Reduce $118,25 to the denominations of Pennsylvania currency.

$$\begin{array}{r} 118,25 \\ 7,5 \\ \hline 59125 \\ 82775 \\ \hline 20\,|\,886,875 \end{array}$$

£44 6s. and ,875 of a shilling, or 10 d. 2 qrs.

Ans. £44 6 s. 10 d. 2 qrs.

5. Reduce $118,25 to the denominations of N. Y. currency.

$$\begin{array}{r} 118,25 \\ 8 \\ \hline 20\,|\,94'6,00 \end{array}$$

Ans. £47 - 6 s.

6. Reduce $118,25 to the denominations of Georgia currency.

Ans. £27 11 s. 10 d.

TO REDUCE ONE CURRENCY TO THE PAR, OR EQUALITY OF ANOTHER.

Reduce the given sum to federal money, and then multiply it by the number of shillings (of the required currency) that make a dollar, and the product will be shillings and decimal parts of a shilling, and may be reduced to pounds, by being divided by 20.

1. Reduce £12 N. Y. currency, to pounds N. E. currency.

12
20

8|240

30 federal money, or $30.

30
6 s. N. E.=1 dollar.

20|18'0 s.

Ans. £9.

2. Reduce £73 13 s. 9 d. N. E. currency to N. Y. currency. *Ans.* £98 5 s.

3. Reduce £16 15 s. N. E. currency to English money. *Ans.* £12 11 s. 3 d.

4. Reduce £27 17 s. 8 d. 3 qrs. English money to N. Y. currency.
Ans. £49 11 s. 6$\frac{2}{3}$ d.

5. Reduce £98 5 s. N. Y. currency to N. E. currency.
Ans. £73 13 s. 9 d.

XI. Dominical Letter.

TO FIND THE DOMINICAL LETTER FOR ANY YEAR.

Set down the date of the year for which you wish to find the dominical letter. To which add one fourth of itself, disregarding the remainder. Divide the amount by 7, and subtract the remainder from 7, if the date of the year be prior to 1800; if later, subtract from 8; and the remainder will be the dominical letter, reckoning from A *toward* G.

1. Find the dominical letter for 1778.

4|1778

444

7|2222 - 3

317

7—3=4. The fourth letter from A is D; therefore D is the dominical letter for 1778.

2. Find the dominical letter for 1841.

$$\begin{array}{r|l} 4 & 1841 \\ \hline & 460 \\ 7 & 2300 - 5 \\ \hline & 328 \end{array}$$

8—5=3. The third letter from A is C. *Ans.* C.

3. Find the dominical letters for 1840.

$$\begin{array}{r|l} 4 & 1840 \\ \hline & 460 \\ 7 & 2300 - 4 \\ \hline & 328 \end{array}$$

The fourth letter from A is D. Therefore D is the dominical letter for 1840, after Feb. 24. As 1840 is leap year, it has two dominical letters, one for January and to the 24th of February, and the other for the remainder of the year. To find the dominical letters for leap year, ascertain what would be the dominical letter for that year, as a common year, and this letter will be the dominical letter after Feb. 24, and the first letter following it will be the dominical letter for January and to the 24th of February. Therefore the dominical letters for 1840 are D and E.

4. Find the dominical letters for 1896. *Ans.* D and E.

5. Find the dominical letter for 1786. *Ans.* A.

6. Find the dominical letter for 1899. *Ans.* A.

TO FIND THE DAY OF THE WEEK, ON WHICH ANY GIVEN DAY OF THE MONTH WILL OCCUR.

Find the dominical letter for the given year; then ascertain what letter begins the month in which the required day is found. Reckon the given number of days from the first day of the month, and you will have the required day.

1. President Harrison was born Feb. 9, 1778; on what day of the week was his birth day?

First find the dominical letter for 1778, which is D. D also stands for the first day of February. Reckon nine days from Sunday, and we have the required day, which was Monday. Ans. Monday.

The following couplet will be of service to the scholar, as the first letter of each word stands for the first day of each month, in the order in which they are placed.

At Dover Dwells George Brown, Esquire,
Good Carlos Finch, And David Fryer.

2. On what day of the week did Pres. Harrison die, it being the 4th of April, 1841? *Ans.* Sunday.

3. On what day of the week was Gen. Hamilton killed in a duel, by Aaron Burr, it being July 11, 1804?
Ans. Wednesday.

4. On what day of the week did the 4th of July come, in 1775? *Ans.* Tuesday.

5. On what day of the week was you born?

XII. To reduce Vulgar Fractions to Decimals.

RULE.—*Annex ciphers to the numerator, and divide by the denominator.*

1. Reduce $\frac{3}{4}$ to a decimal.

$$4|3,00$$

Ans. ,75

2. Reduce $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, to decimals. *Ans.* ,5, ,25, ,125.

3. Reduce $\frac{1}{7}$, $\frac{1}{9}$, $\frac{1}{11}$, $\frac{1}{13}$, to decimals.
Ans. ,142857$\frac{1}{7}$, ,111111$\frac{1}{9}$, ,090909$\frac{1}{11}$, ,076923$\frac{1}{13}$.

In the last example, there was a remainder in each case, and would have been, had the division been carried to an indefinite extent. Instead of dropping this remainder, I have chosen to express it in the form of a vulgar fraction. In all practical cases, the decimal, carried to four or five places, or even three, in federal money, is sufficiently exact. But in the examples wrought in this work, the true answer will always be given to the very last fraction.

XIII. To reduce different Denominations to one and the Decimal of one.

RULE.—*Beginning with the lowest denomination, divide by that number of the lowest denomination which makes a unit of the next higher. Annex the quotient to the next higher denomination, and proceed as before, until you have reduced the given denominations to the decimal of the required one.*

1. Reduce £6 11 s. 11 d. 3 qrs. to shillings and decimal of a shilling.

$$\begin{array}{l} 6\ \ 11\ \ 11\ \ 3 \\ \underline{20} \\ 131 \end{array} \qquad \begin{array}{r|l} 4 & 3,0 \\ \hline & ,75 \end{array} \qquad \begin{array}{r|l} 12 & 11,75 \\ \hline & ,97916\tfrac{2}{3} \end{array}$$

Ans. 131,97916$\tfrac{2}{3}$.

I multiplied £6 by 20 s. to bring the pounds into shillings, and to these shillings I added 11 s. I then divided 3 qrs. by 4 d. to reduce the farthings to the decimal of a penny. I annexed this decimal of a penny to 11 d. and divided by 12 d. to reduce them to the decimal of a shilling. This decimal of a shilling I annexed to shillings.

2. Reduce £8 11 s. 8 d. 3 d. to pounds and decimal of a pound.

$$\begin{array}{r|l} 4 & 3,00 \\ \hline 12 & 8,75 \\ \hline 20 & 11,72916\tfrac{2}{3} \\ \hline & ,586453\tfrac{1}{3} \end{array}$$

Ans. 8,586453$\tfrac{1}{3}$.

3. Reduce 4 cwt. 3 qrs. 16 lb. 8 oz. to cwt. and the decimal of a cwt. *Ans.* 4,915

4. Reduce 13 hours, 40 minutes, and 35 seconds, to the decimal of a year. *Ans.* ,00156016$\tfrac{11848}{39[illegible]47}$.

5. Reduce 19 s. 9 d. to the decimal of a pound. *Ans.* ,9875.

6. Reduce 9 s. 13° 25′ to the decimal of a circle. *Ans.* ,787$\tfrac{22}{[illegible]}$.

7. Reduce 8 cord-feet, 7 feet board measure, 5 primes, 6 inches, to the decimal of a cord.

Ans. $1,00485568\frac{249}{432}$.

8. Reduce 15 yrs. 19 d. 11 h. 37 m. 45 s. to years, and the decimal of a year. *Ans.* $15,0533\frac{36123}{78894}$.

9. Reduce 475047465 seconds to years and the decimal of a year. *Ans.* $15,0533\frac{36123}{78894}$.

XIV. To reduce the Decimal of a higher Denomination to Units of a lower Denomination.

RULE.—*Multiply the given decimal by that number of the next lower denomination which makes a unit of the given denomination, and point off your decimals from the right hand of the product.*

1. Reduce 4,915 cwt. to units of a lower denomination.

```
 4,915
     4
 -----
 3,660
    25
 -----
  3300
 1320
 ------
16,500
    16
 ------
  3000
  500
 ------
 8,000
```

Ans. 4 cwt. 3 qrs. 16 lb. 8 oz.

2. Reduce £ ,9875 to shillings and pence.
Ans. 19 s. 9 d.

3. Reduce £ ,6625 to shillings and pence.
Ans. 13 s. 3 d.

4. Reduce £ ,6875 to shillings and pence.
Ans. 13 s. 9 d.

5. Reduce £ ,7 to shillings. *Ans.* 14 s.

6. Reduce £ ,015625 to pence and farthings.
Ans. 3 d. 3 qrs.

7. Reduce ,875 of a tun of wine to pipes, hogsheads, gallons, and quarts.
Ans. 1 p. 1 hhd. 31 gal. 2 qt.

XV. Decimals.

The first figure at the left hand of the decimal point is a *unit*, and the first figure at the right hand of the same point is *tenths*. Thus 1,1 represent one and one tenth. It is plain that the unit figure may represent any thing you may please to name, money, weight, measure, &c. But whatever denomination you give to the unit, the first figure at the right of the decimal point stands for one tenth of the unit of that denomination. If the unit stand for one dollar, then the ,1 represents a tenth part of a dollar. If the unit stand for one pound, then the ,1 represents one tenth of a pound. Cents and mills are decimal parts of a dollar. Ounces and drams may be reduced to the decimal of a pound; quarts and pints to the decimal of a bushel; hours and minutes to the decimal of a day; inches to the decimal of a foot. In one word, every unit has the same parts in a decimal form, that a dollar has. Therefore, every thing ciphers like money. If ,25 are a fourth part of a dollar, in reckoning money, the same expression may represent a fourth part of any unit of any other denomination. $,125 represent one eighth of a dollar. But ,125 represent one eighth of any unit. To make the scholar familiar with decimal parts, several pages are filled with Tables.

TABLES SHOWING THE ALIQUOT PARTS OF A UNIT.

Half.		$\frac{1}{2}$	$= ,5$
3ds.		$\frac{1}{3}$	$= ,33\frac{1}{3}$
		$\frac{2}{3}$	$= ,66\frac{2}{3}$
4ths.		$\frac{1}{4}$	$= ,25$
	$\frac{1}{2} =$	$\frac{2}{4}$	$= ,5$
		$\frac{3}{4}$	$= ,75$
5ths.		$\frac{1}{5}$	$= ,2$
		$\frac{2}{5}$	$= ,4$
		$\frac{3}{5}$	$= ,6$
		$\frac{4}{5}$	$= ,8$
6ths.		$\frac{1}{6}$	$= ,16\frac{2}{3}$
	$\frac{1}{3} =$	$\frac{2}{6}$	$= ,33\frac{1}{3}$
	$\frac{1}{2} =$	$\frac{3}{6}$	$= ,5$
	$\frac{2}{3} =$	$\frac{4}{6}$	$= ,66\frac{2}{3}$
		$\frac{5}{6}$	$= ,83\frac{1}{3}$
7ths.		$\frac{1}{7}$	$= ,142857\frac{1}{7}$
		$\frac{2}{7}$	$= ,285714\frac{2}{7}$
		$\frac{3}{7}$	$= ,428571\frac{3}{7}$
		$\frac{4}{7}$	$= ,571428\frac{4}{7}$
		$\frac{5}{7}$	$= ,714285\frac{5}{7}$
		$\frac{6}{7}$	$= ,857142\frac{6}{7}$
8ths.		$\frac{1}{8}$	$= ,125$
	$\frac{1}{4} =$	$\frac{2}{8}$	$= ,25$
		$\frac{3}{8}$	$= ,375$
	$\frac{1}{2} =$	$\frac{4}{8}$	$= ,5$
		$\frac{5}{8}$	$= ,625$
	$\frac{3}{4} =$	$\frac{6}{8}$	$= ,75$
		$\frac{7}{8}$	$= ,875$
9ths.		$\frac{1}{9}$	$= ,111\frac{1}{9}$
		$\frac{2}{9}$	$= ,222\frac{2}{9}$
	$\frac{1}{3} =$	$\frac{3}{9}$	$= ,333\frac{3}{9}$
		$\frac{4}{9}$	$= ,444\frac{4}{9}$
		$\frac{5}{9}$	$= ,555\frac{5}{9}$
	$\frac{2}{3} =$	$\frac{6}{9}$	$= ,666\frac{6}{9}$
		$\frac{7}{9}$	$= ,777\frac{7}{9}$
		$\frac{8}{9}$	$= ,888\frac{8}{9}$

12ths.		$\frac{1}{12}$	$= ,08\frac{1}{3}$
	$\frac{1}{6} =$	$\frac{2}{12}$	$= ,16\frac{2}{3}$
	$\frac{1}{4} =$	$\frac{3}{12}$	$= ,25$
	$\frac{1}{3} =$	$\frac{4}{12}$	$= ,33\frac{1}{3}$
		$\frac{5}{12}$	$= ,41\frac{2}{3}$
	$\frac{1}{2} =$	$\frac{6}{12}$	$= ,5$
		$\frac{7}{12}$	$= ,58\frac{1}{3}$
	$\frac{2}{3} =$	$\frac{8}{12}$	$= ,66\frac{2}{3}$
	$\frac{3}{4} =$	$\frac{9}{12}$	$= ,75$
	$\frac{5}{6} =$	$\frac{10}{12}$	$= ,83\frac{1}{3}$
		$\frac{11}{12}$	$= ,91\frac{2}{3}$
16ths.		$\frac{1}{16}$	$= ,0625$
	$\frac{1}{8} =$	$\frac{2}{16}$	$= ,125$
		$\frac{3}{16}$	$= ,1875$
	$\frac{1}{4} =$	$\frac{4}{16}$	$= ,25$
		$\frac{5}{16}$	$= ,3125$
	$\frac{3}{8} =$	$\frac{6}{16}$	$= ,375$
		$\frac{7}{16}$	$= ,4375$
	$\frac{1}{2} =$	$\frac{8}{16}$	$= ,5$
		$\frac{9}{16}$	$= ,5625$
	$\frac{5}{8} =$	$\frac{10}{16}$	$= ,625$
		$\frac{11}{16}$	$= ,6875$
	$\frac{3}{4} =$	$\frac{12}{16}$	$= ,75$
		$\frac{13}{16}$	$= ,8125$
	$\frac{7}{8} =$	$\frac{14}{16}$	$= ,875$
		$\frac{15}{16}$	$= ,9375$
20ths.		$\frac{1}{20}$	$= ,05$
		$\frac{2}{20}$	$= ,1$
		$\frac{3}{20}$	$= ,15$, &c.
25ths.		$\frac{1}{25}$	$= ,04$
		$\frac{2}{25}$	$= ,08$
		$\frac{3}{25}$	$= ,12$
		$\frac{4}{25}$	$= ,16$, &c.
32d.		$\frac{1}{32}$	$= ,03125$
		$\frac{3}{32}$	$= ,09375$
		$\frac{5}{32}$	$= ,15625$
		$\frac{7}{32}$	$= ,21875$

Dollar.	Dollar.	Dollar.	Cents.	Cents.	N. Y.
$\frac{.25}{8}$	$\frac{.5}{16}$	$\frac{1}{32}$	,03125	$3\frac{1}{8}$	3 d.
$\frac{.5}{8}$	$\frac{1}{16}$	$\frac{2}{32}$	,0625	$6\frac{1}{4}$	6 d.
$\frac{.75}{8}$	$\frac{1.5}{16}$	$\frac{3}{32}$	,09375	$9\frac{3}{8}$	9 d.
$\frac{1}{8}$	$\frac{2}{16}$	$\frac{4}{32}$	,125	$12\frac{1}{2}$	1 s.
$\frac{1.25}{8}$	$\frac{2.5}{16}$	$\frac{5}{32}$	,15625	$15\frac{5}{8}$	1 s. 3 d.
$\frac{1.5}{8}$	$\frac{3}{16}$	$\frac{6}{32}$	,1875	$18\frac{3}{4}$	1 s. 6 d.
$\frac{1.75}{8}$	$\frac{3.5}{16}$	$\frac{7}{32}$	,21875	$21\frac{7}{8}$	1 s. 9 d.
$\frac{2}{8}$	$\frac{4}{16}$	$\frac{8}{32}$	,25	25	2 s.
$\frac{2.25}{8}$	$\frac{4.5}{16}$	$\frac{9}{32}$	,28125	$28\frac{1}{8}$	2 s. 3 d.
$\frac{2.5}{8}$	$\frac{5}{16}$	$\frac{10}{32}$	,3125	$31\frac{1}{4}$	2 s. 6 d.
$\frac{2.75}{8}$	$\frac{5.5}{16}$	$\frac{11}{32}$	,34375	$34\frac{3}{8}$	2 s. 9 d.
$\frac{3}{8}$	$\frac{6}{16}$	$\frac{12}{32}$	,375	$37\frac{1}{2}$	3 s.
$\frac{3.25}{8}$	$\frac{6.5}{16}$	$\frac{13}{32}$	,40625	$40\frac{5}{8}$	3 s. 3d.
$\frac{3.5}{8}$	$\frac{7}{16}$	$\frac{14}{32}$	,4375	$43\frac{3}{4}$	3 s. 6 d.
$\frac{3.75}{8}$	$\frac{7.5}{16}$	$\frac{15}{32}$	,46875	$46\frac{7}{8}$	3 s. 9 d.
$\frac{4}{8}$	$\frac{8}{16}$	$\frac{16}{32}$	,5	50	4 s.
$\frac{4.25}{8}$	$\frac{8.5}{16}$	$\frac{17}{32}$	,53125	$53\frac{1}{8}$	4 s. 3 d.
$\frac{4.5}{8}$	$\frac{9}{16}$	$\frac{18}{32}$	,5625	$56\frac{1}{4}$	4 s. 6 d.
$\frac{4.75}{8}$	$\frac{9.5}{16}$	$\frac{19}{32}$	,59375	$59\frac{3}{8}$	4 s. 9 d.
$\frac{5}{8}$	$\frac{10}{16}$	$\frac{20}{32}$	,625	$62\frac{1}{2}$	5 s.
$\frac{5.25}{8}$	$\frac{10.5}{16}$	$\frac{21}{32}$	,65625	$65\frac{5}{8}$	5 s. 3 d.
$\frac{5.5}{8}$	$\frac{11}{16}$	$\frac{22}{32}$	,6875	$68\frac{3}{4}$	5 s. 6 d.
$\frac{5.75}{8}$	$\frac{11.5}{16}$	$\frac{23}{32}$	,71875	$71\frac{7}{8}$	5 s. 9 d.
$\frac{6}{8}$	$\frac{12}{16}$	$\frac{24}{32}$	,75	75	6 s.
$\frac{6.25}{8}$	$\frac{12.5}{16}$	$\frac{25}{32}$	,78125	$78\frac{1}{8}$	6 s. 3 d.
$\frac{6.5}{8}$	$\frac{13}{16}$	$\frac{26}{32}$	,8125	$81\frac{1}{4}$	6 s. 6 d.
$\frac{6.75}{8}$	$\frac{13.5}{16}$	$\frac{27}{32}$	,84375	$84\frac{3}{8}$	6 s. 9 d.
$\frac{7}{8}$	$\frac{14}{16}$	$\frac{28}{32}$	,875	$87\frac{1}{2}$	7 s.
$\frac{7.25}{8}$	$\frac{14.5}{16}$	$\frac{29}{32}$	,90625	$90\frac{5}{8}$	7 s. 3 d.
$\frac{7.5}{8}$	$\frac{15}{16}$	$\frac{30}{32}$	,9375	$93\frac{3}{4}$	7 s. 6 d.
$\frac{7.75}{8}$	$\frac{15.5}{16}$	$\frac{31}{32}$	,96875	$96\frac{7}{8}$	7 s. 9 d.

Dollar.	Dollar.	Dollar.	Cents.	N. E.
$\frac{.25}{6}$ =	$\frac{.5}{12}$ =	$\frac{1}{24}$ =	,04$\frac{1}{6}$	= 3 d.
$\frac{.5}{6}$ =	$\frac{1}{12}$ =	$\frac{2}{24}$ =	,08$\frac{1}{3}$	= 6 d.
$\frac{.75}{6}$ =	$\frac{1.5}{12}$ =	$\frac{3}{24}$ =	,125	= 9 d.
$\frac{1}{6}$ =	$\frac{2}{12}$ =	$\frac{4}{24}$ =	,16$\frac{2}{3}$	= 1 s.
$\frac{1.25}{6}$ =	$\frac{2.5}{12}$ =	$\frac{5}{24}$ =	,20$\frac{5}{6}$	= 1 s. 3 d.
$\frac{1.5}{6}$ =	$\frac{3}{12}$ =	$\frac{6}{24}$ =	,25	= 1 s. 6 d.
$\frac{1.75}{6}$ =	$\frac{3.5}{12}$ =	$\frac{7}{24}$ =	,29$\frac{1}{6}$	= 1 s. 9 d.
$\frac{2}{6}$ =	$\frac{4}{12}$ =	$\frac{8}{24}$ =	,33$\frac{1}{3}$	= 2 s.
$\frac{2.25}{6}$ =	$\frac{4.5}{12}$ =	$\frac{9}{24}$ =	,375	= 2 s. 3 d.
$\frac{2.5}{6}$ =	$\frac{5}{12}$ =	$\frac{10}{24}$ =	,41$\frac{2}{3}$	= 2 s. 6 d.
$\frac{2.75}{6}$ =	$\frac{5.5}{12}$ =	$\frac{11}{24}$ =	,45$\frac{5}{6}$	= 2 s. 9 d.
$\frac{3}{6}$ =	$\frac{6}{12}$ =	$\frac{12}{24}$ =	,5	= 3 s.
$\frac{3.25}{6}$ =	$\frac{6.5}{12}$ =	$\frac{13}{24}$ =	,54$\frac{1}{6}$	= 3 s. 3 d.
$\frac{3.5}{6}$ =	$\frac{7}{12}$ =	$\frac{14}{24}$ =	,58$\frac{1}{3}$	= 3 s. 6 d.
$\frac{3.75}{6}$ =	$\frac{7.5}{12}$ =	$\frac{15}{24}$ =	,625	= 3 s. 9 d.
$\frac{4}{6}$ =	$\frac{8}{12}$ =	$\frac{16}{24}$ =	,66$\frac{2}{3}$	= 4 s.
$\frac{4.25}{6}$ =	$\frac{8.5}{12}$ =	$\frac{17}{24}$ =	,70625	= 4 s. 3 d.
$\frac{4.5}{6}$ =	$\frac{9}{12}$ =	$\frac{18}{24}$ =	,75	= 4 s. 6 d.
$\frac{4.75}{6}$ =	$\frac{9.5}{12}$ =	$\frac{19}{24}$ =	,79$\frac{1}{6}$	= 4 s. 9 d.
$\frac{5}{6}$ =	$\frac{10}{12}$ =	$\frac{20}{24}$ =	,83$\frac{1}{3}$	= 5 s.
$\frac{5.25}{6}$ =	$\frac{10.5}{12}$ =	$\frac{21}{24}$ =	,875	= 5 s. 3 d.
$\frac{5.5}{6}$ =	$\frac{11}{12}$ =	$\frac{22}{24}$ =	,91$\frac{2}{3}$	= 5 s. 6 d.
$\frac{5.75}{6}$ =	$\frac{11.5}{12}$ =	$\frac{23}{24}$ =	,95$\frac{5}{6}$	= 5 s. 9 d.

Dollar.		Dollar.		Dollar.		Cents.		Canada.
$\frac{.25}{5}$	=	$\frac{.5}{10}$	=	$\frac{1}{20}$	=	,05	=	3 d.
$\frac{.5}{5}$	=	$\frac{1}{10}$	=	$\frac{2}{20}$	=	,1	=	6 d.
$\frac{.75}{5}$	=	$\frac{1.5}{10}$	=	$\frac{3}{20}$	=	,15	=	9 d.
$\frac{1}{5}$	=	$\frac{2}{10}$	=	$\frac{4}{20}$	=	,2	=	1 s.
$\frac{1.25}{5}$	=	$\frac{2.5}{10}$	=	$\frac{5}{20}$	=	,25	=	1 s. 3 d.
$\frac{1.5}{5}$	=	$\frac{3}{10}$	=	$\frac{6}{20}$	=	,3	=	1 s. 6 d.
$\frac{1.75}{5}$	=	$\frac{3.5}{10}$	=	$\frac{7}{20}$	=	, 35	=	1 s. 9 d.
$\frac{2}{5}$	=	$\frac{4}{10}$	=	$\frac{8}{20}$	=	,40	=	2 s.
$\frac{2.25}{5}$	=	$\frac{4.5}{10}$	=	$\frac{9}{20}$	=	,45	=	2 s. 3 d.
$\frac{2.5}{5}$	=	$\frac{5}{10}$	=	$\frac{10}{20}$	=	,5	=	2 s. 6 d.
$\frac{2.75}{5}$	=	$\frac{5.5}{10}$	=	$\frac{11}{20}$	=	,55	=	2 s. 9 d.
$\frac{3}{5}$	=	$\frac{6}{10}$	=	$\frac{12}{20}$	=	,6	=	3 s.
$\frac{3.25}{5}$	=	$\frac{6.5}{10}$	=	$\frac{13}{20}$	=	,65	=	3 s. 3 d.
$\frac{3.5}{5}$	=	$\frac{7}{10}$	=	$\frac{14}{20}$	=	,7	=	3 s. 6 d.
$\frac{3.75}{5}$	=	$\frac{7.5}{10}$	=	$\frac{15}{20}$	=	,75	=	3 s. 9 d.
$\frac{4}{5}$	=	$\frac{8}{10}$	=	$\frac{16}{20}$	=	,8	=	4 s.
$\frac{4.25}{5}$	=	$\frac{8.5}{10}$	=	$\frac{17}{20}$	=	,85	=	4 s. 3 d.
$\frac{4.5}{5}$	=	$\frac{9}{10}$	=	$\frac{18}{20}$	=	,9	=	4 s. 6 d.
$\frac{4.75}{5}$	=	$\frac{9.5}{10}$	=	$\frac{19}{20}$	=	,95	=	4 s. 9 d.

TABLE

SHOWING THE VALUE OF NUMBERS ACCORDING TO THEIR LOCATION IN RELATION TO THE DECIMAL POINT.

	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.
1	10000,	1000,	100,	10,	1,	,1	,01	,001
$\frac{1}{2}$	5000,	500,	50,	5,	,5	,05	,005	,0005
$\frac{1}{3}$	3333,33$\frac{1}{3}$	333,33$\frac{1}{3}$	33,333$\frac{1}{3}$	3,333$\frac{1}{3}$	,333$\frac{1}{3}$	,0333$\frac{1}{3}$	,00333$\frac{1}{3}$	,000333$\frac{1}{3}$
$\frac{1}{4}$	2500,	250,	25,	2,5	,25	,025	,0025	,00025
$\frac{1}{5}$	2000,	200,	20,	2,	,2	,02	,002	,0002
$\frac{1}{6}$	1666,66$\frac{2}{3}$	166,666$\frac{2}{3}$	16,6666$\frac{2}{3}$	1,66$\frac{2}{3}$	,16$\frac{2}{3}$	,01666$\frac{2}{3}$	,00166$\frac{2}{3}$	,0001666$\frac{2}{3}$
$\frac{1}{7}$	1428,57$\frac{1}{7}$	142,857$\frac{1}{7}$	14,2857$\frac{1}{7}$	1,42857$\frac{1}{7}$	,142857$\frac{1}{7}$	,0142857$\frac{1}{7}$	,00142857$\frac{1}{7}$	,000142857$\frac{1}{7}$
$\frac{1}{8}$	1250,	125,	12,5	1,25	,125	,0125	,00125	,000125
$\frac{1}{9}$	1111,11$\frac{1}{9}$	111,1111$\frac{1}{9}$	11,111$\frac{1}{9}$	1,111111$\frac{1}{9}$	,1111111$\frac{1}{9}$	,011111$\frac{1}{9}$	,001111$\frac{1}{9}$	,0001111$\frac{1}{9}$
$\frac{1}{10}$	1000,	100,	10,	1,	,1	,01	,001	,0001
$\frac{1}{11}$	909,09$\frac{1}{11}$	90,909$\frac{1}{11}$	9,09$\frac{1}{11}$	,909$\frac{1}{11}$	,0909$\frac{1}{11}$	,009$\frac{1}{11}$	,0009$\frac{1}{11}$	,00009$\frac{1}{11}$

TABLE *continued.*

	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.
$\frac{1}{12}$	833,33$\frac{1}{3}$	83,3333$\frac{1}{3}$	8,3333$\frac{1}{3}$	,8333$\frac{1}{3}$	,08333$\frac{1}{3}$	,00833$\frac{1}{3}$	,00083$\frac{1}{3}$	,000083$\frac{1}{3}$
$\frac{1}{16}$	625,	62,5	6,25	,625	,0625	,00625	,000625	,0000625
$\frac{1}{20}$	500,	50,	5,	,5	,05	,005	,0005	,00005
$\frac{1}{25}$	400,	40,	4,	,4	,04	,004	,0004	,00004
$\frac{1}{32}$	312,5	31,25	3,125	,3125	,03125	,003125	,0003125	,00003125
$\frac{1}{40}$	250,	25,	2,5	,25	,025	,0025	,00025	,000025
$\frac{1}{64}$	156,25	15,625	1,5625	,15625	,015625	,0015625	,00015625	,000015625
$\frac{3}{8}$	3750,	375,	37,5	3,75	,375	,0375	,00375	,000375
$\frac{5}{8}$	6250,	625,	62,5	6,25	,625	,0625	,00625	,000625
$\frac{7}{8}$	8750,	875,	87,5	8,75	,875	,0875	,00875	,000875
$\frac{3}{16}$	1875,	187,5	18,75	1,875	,1875	,01875	,001875	,0001875
$\frac{5}{16}$	3125,	312,5	31,25	3,125	,3125	,03125	,003125	,0003125

TABLE *continued.*

	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.
$\frac{7}{16}$	4375,	437,5	43,75	4,375	,4375	,04375	,004375	,0004375
$\frac{9}{16}$	5625,	562,5	56,25	5,625	,5625	,05625	,005625	,0005625
$\frac{11}{16}$	6875,	687,5	68,75	6,875	,6875	,06875	,006875	,0006875
$\frac{13}{16}$	8125,	812,5	81,25	8,125	,8125	,08125	,008125	,0008125
$\frac{15}{16}$	9375,	937,5	93,75	9,375	,9375	,09375	,009375	,0009375
$\frac{3}{32}$	937,5	93,75	9,375	,9375	,09375	,009375	,0009375	,00009375
$\frac{5}{32}$	1562,5	156,25	15,625	1,5625	,15625	,015625	,0015625	,00015625
$\frac{7}{32}$	2187,5	218,75	21,875	2,1875	,21875	,021875	,0021875	,00021875
$\frac{9}{32}$	2812,5	281,25	28,125	2,8125	,28125	,028125	,0028125	,00028125
$\frac{11}{32}$	3437,5	343,75	34,375	3,4375	,34375	,034375	,0034375	,00034375
$\frac{13}{32}$	4062,5	406,25	40,625	4,0625	,40625	,040625	,0040625	,00040625
$\frac{15}{32}$	4687,5	468,75	46,875	4,6875	,46875	,046875	,0046875	,00046875

NOTE.—To multiply by any numbers in this table, and all other similar numbers, multiply by the numerator, and divide by the denominator.

To divide by the same numbers, divide by the numerator, and multiply by the denominator of the vulgar fraction in the left-hand column, and point off by the decimal rules.

ADDITION OF DECIMALS.

1. Add $\frac{1}{8}$ of 10; $\frac{1}{8}$ of 1000; $\frac{1}{8}$ of ,01; $\frac{1}{8}$ of 100.

By consulting the *table* showing the value of numbers according to their location, the learner will find that

$$\begin{array}{lcr} \frac{1}{8} \text{ of } 1000 & = & 125, \\ \frac{1}{8} \text{ of } 100 & = & 12,5 \\ \frac{1}{8} \text{ of } 10 & = & 1,25 \\ \frac{1}{8} \text{ of } ,01 & = & ,00125 \\ \hline & \textit{Ans.} & 138,75125 \end{array}$$

2. Add $\frac{7}{8}$ of 10 eagles; $\frac{5}{8}$ of 1 dime; $\frac{1}{16}$ of 1 dollar; $\frac{1}{7}$ of 1000 dollars.

$$\begin{array}{lcr} \frac{7}{8} \text{ of 10 eagles} & = & \$87,5 \\ \frac{5}{8} \text{ of 1 dime} & = & ,0625 \\ \frac{1}{16} \text{ of 1 dollar} & = & ,0625 \\ \frac{1}{7} \text{ of 1000 dolls.} & = & 142,8571\frac{3}{7} \\ \hline & \textit{Ans.} & \$230,4821\frac{3}{7} \end{array}$$

3. Add $\frac{1}{8}$ of 1000; $\frac{1}{2}$ of 1; $\frac{1}{4}$ of ,1; $\frac{3}{4}$ of ,001; $\frac{3}{8}$ of 100; $\frac{5}{8}$ of ,01; $\frac{7}{8}$ of ,001; $\frac{7}{8}$ of 1000; $\frac{3}{32}$ of 1; and $\frac{1}{64}$ of 1000. *Ans.* 1053,751625.

4. Add $\frac{3}{8}$ of 10000; $\frac{4}{5}$ of 1000; $\frac{7}{8}$ of ,001; $\frac{3}{16}$ of 100; $\frac{5}{16}$ of ,01; $\frac{15}{16}$ of 1000; $\frac{13}{16}$ of ,1; $\frac{11}{16}$ of 10; $\frac{9}{16}$ of 10; $\frac{19}{64}$ of 1000; $\frac{11}{32}$ of ,01; $\frac{19}{32}$ of 10000, and $\frac{29}{32}$ of 100. *Ans.* 11703,2136875.

5. Add 7 bushels, 10 qts.; 11 bushels, 22 qts.; 15 bushels, 30 qts.; and 18 qts. The learner should remember that a quart is $\frac{1}{32}$ of a bushel, or ,03125 of a bushel. *Ans.* 35,5.

6. To $\frac{1}{2}$ of a shilling add $\frac{3}{4}$ of a penny.

$$\begin{array}{lcr} \frac{3}{4} \text{ of a penny} & = & ,75 \\ \frac{1}{2} \text{ of a shilling} & = & 6, \\ \hline & \textit{Ans.} & 6,75 \text{ pence.} \end{array}$$

7. To $\frac{3}{4}$ of a shilling add $\frac{7}{8}$ of 10 pence.

$$\begin{array}{lcr} \frac{3}{4} \text{ of a shilling} & = & 9, \text{ d.} \\ \frac{7}{8} \text{ of 10 pence} & = & 8,75 \\ \hline & \textit{Ans.} & 17,75 \text{ pence, or 1 s. } 5\frac{3}{4} \text{ d.} \end{array}$$

8. To 3 d. add $\frac{4}{5}$ of a shilling. *Ans.* 1,05 s.

9. If a man has due to him 83 bushels and 3 pecks of wheat; 19 bushels and one peck of rye; 36 bushels and 9 qts. of corn; 15 bushels and 22 qts. of oats; 7 bushels and 11 qts. of barley; and 13 bushels and 13 qts. of India wheat; how much grain is due to him?

Ans. 175,71875 bushels, or 175 bushels, 23 qts.

10. Suppose a man owes 6 debts—the 1st, \$83,75; the 2d, \$19,25; the 3d, \$36,28$\frac{1}{8}$; the 4th, \$15,68$\frac{3}{4}$; the 5th, \$7,34$\frac{3}{8}$; and the 6th, \$13,40$\frac{5}{8}$. What is the amount of his debts? *Ans.* \$175,71875.

The same figures are employed in expressing the whole numbers and decimals in the 9th and 10th examples, and, of course, the answers are the same in both cases.

11. Add $9\frac{1}{16}$, $8\frac{5}{16}$, $4\frac{3}{16}$, $7\frac{10}{16}$. *Ans.* 29,1875, or $29\frac{3}{16}$.

12. Add $\frac{1}{64}$ of 1000000, $\frac{1}{64}$ of ,000001, and $\frac{1}{64}$ of 1.

Ans. 15625,015625015625.

SUBTRACTION OF DECIMALS.

1. From 8 pounds and 12 ounces take 6 pounds and 8 ounces.

12 oz. = ,75 of a pound.
8 oz. = ,5 " "

8,75
6,5

Ans. lb. 2,25, or $2\frac{1}{4}$ pounds, or 2 lbs. 4 oz.

2. A merchant received 3 boxes of butter from his customers. The butter and boxes weighed as follows: 7 pounds and 3 oz.; 9 pounds and 6 oz.; 11 pounds and 9 oz. One box weighed 1 pound and 5 oz.; another, 1 pound and 7 ounces; and the third, 2 pounds and 2 ounces. What was the weight of the butter?

7,1875 = 7 pounds and 3 oz.
9,375 = 9 " 6 oz.
11,5625 = 11 " 9 oz.

28,1250, the weight of the butter and boxes.

1,3125 = 1 pound and 5 oz.
1,4375 = 1 " 7 "
2,125 = 2 " 2 ".

4,8750, the weight of the boxes.

28,125
4,875

Ans. lb. 23,250, the weight of the butter.

3. From $43\frac{1}{16}$ yards take $5\frac{3}{8}$, $7\frac{1}{8}$, and $13\frac{5}{16}$ yards. *Ans.* 18,125.

4. From 7 cords, and 7 cord feet of wood, take 3 cords, and 2 cord feet. *Ans.* 4,625, or $4\frac{5}{8}$.

5. From 7 pounds and 14 oz. take 3 pounds and 2 oz. *Ans.* 4,75, or 4 lb. 12 oz.

6. From half a shilling take $\frac{3}{4}$ of a penny. *Ans.* 5,25 d.

7. From $\frac{5}{6}$ of a dollar take $\frac{3,5}{6}$ of a dollar.

$\frac{5}{6} = ,833\frac{1}{3}$ Or thus: $\frac{5}{6} = 5$ s.
$\frac{3,5}{6} = ,583\frac{1}{3}$ $\frac{3,5}{6} = 3,5$

Ans. ,25 *Ans.* 1,5 s. = ,25

8. From 15 s. take 7 s. and 9 d. *Ans.* 7,25 s.

9. From 17 s. 3 d. take 14 s. 9 s. *Ans.* 2,5 s., or 2 s. 6 d. = ,41$\frac{2}{3}$ cts.

10. From 9 cents take $\frac{7}{8}$ of a dime. *Ans.* ,0025 = $\frac{1}{4}$ of a cent.

11. From 7 units take $\frac{5}{8}$ of 10 units. *Ans.* ,75.

12. From 10 units take $\frac{7}{8}$ of 10 units. *Ans.* 1,25.

13. From $\frac{5}{16}$ of 10 units take $\frac{7}{8}$ of ,01. *Ans.* 3,11625.

14. From $\frac{15}{16}$ of 1, take $\frac{15}{16}$ of ,01. *Ans.* ,928125.

15. From 39 pounds, 11 oz. take 21 pounds and 15 oz. *Ans.* 17,75.

16. From $\frac{3}{16}$ of 100, $\frac{5}{16}$ of 1000, $\frac{15}{16}$ of 10, $\frac{9}{16}$ of ,01, $\frac{1}{8}$ of 1000, $\frac{3}{8}$ of ,001, and ,369, take 341. *Ans.* 125.

17. From $\frac{18}{20}$ of 10000, $\frac{13}{20}$ of ,0001, $\frac{15}{40}$ of 1000, $\frac{25}{40}$ of ,001, $\frac{7}{8}$ of 10, $\frac{5}{8}$ of ,00000001, $\frac{11}{32}$ of ,001, $\frac{14\frac{3}{4}}{20}$ of ,000001,

and $\frac{15}{32}$ of 10000000, take $\frac{15\frac{3}{4}}{20}$ of 1000, $\frac{11\frac{1}{4}}{20}$ of 100, $\frac{7\frac{1}{2}}{20}$ of ,0001, $\frac{19}{40}$ of 1000000, $\frac{11}{40}$ of ,0000001, and $\frac{19}{32}$ of 100.

Ans. 4221475,62624696625

MULTIPLICATION OF DECIMALS.

Multiplication is repeating the value of the multiplicand as many times as the multiplier contains a unit. If the multiplier is a unit, or 1, the multiplicand is repeated *once.* If the multiplier is less than a unit, then a *part* only of the multiplicand is repeated. If the multiplier is ,5, or $\frac{1}{2}$, then one half of the multiplicand is repeated once. If the multiplier is ,25, or $\frac{1}{4}$, one fourth of the multiplicand is repeated once. If the multiplier is ,125, or $\frac{1}{8}$, one eighth of the multiplicand is repeated once.

When the multiplier is ,5, ,25, ,2, ,16$\frac{2}{3}$, ,142857$\frac{1}{7}$, ,125, &c. the answer may be readily obtained by dividing by 2, 4, 5, 6, 7, 8, &c. observing carefully the position of the decimal point.

1. Multiply ,5 by ,5.

```
      ,5      Or thus:  2|,5
      ,5                ----
      --           Ans. ,25
Ans. ,25
```

It will be seen that ,5 of a dollar are fifty cents. Therefore, if ,5 be considered *money*, the answer is 25 cents, or a quarter of a dollar.

2. Multiply ,25 by ,25.

```
       ,25     Or thus:     4|,25
       ,25                   ----
       ---              Ans. ,0625=6¼ cents, or 1/16 [of a unit.
       125
       50
      ----
Ans. ,0625
```

3. Multiply ,125 by ,125.

```
,125          Or thus:   8|,125
,125                      ------
----                 Ans. ,015625 = 1/64 of a unit.
 625
250
125
-------
```

Ans. ,015625 = 1 cent, 5 mills, and $\frac{5}{8}$ of a mill, if we call ,125 money

4. What will 9 pounds and 2 oz. cost, at ,125 of a dollar per pound?

9,125 = 9 pounds and 2 oz.

Then, as 8 pounds cost a dollar, divide by 8, and the quotient will be the answer.

8|9,125

Ans. \$1,140625 = 1 dollar, 14 cts. 0 mill, and $\frac{5}{8}$ of a mill.

5. What will 79 pounds and 13 oz. of sugar cost, at ,16$\frac{2}{3}$ per pound?

See Table. $\frac{13}{16}$ = ,8125.

6|79,8125

Ans. \$13,302083$\frac{1}{3}$ = 13 dollars, 30 cents, 2 mills, and $\frac{1}{12}$ of a mill.

6. What will 68 pounds and 13 oz. of tea cost, at \$,83$\frac{1}{3}$ per pound?

\$,83$\frac{1}{3}$ = $\frac{10}{12}$ of a dollar, and 13 oz. = ,8125 of a pound.

12|68,8125

Ans. \$57,34375

I multiplied by 10, by removing the decimal point one figure to the right.

The same method of abbreviating, as adopted in the foregoing examples, may be employed when the multiplier is a *mixed* number, or a *whole* number. But the decimal point will be removed to the right, one figure in the answer, for *every figure at the left* of the point in the multiplier.

7. What will 9 cords, and 3 cord feet of wood, cost, at $2,5 per cord?

3 cord feet=$\frac{3}{8}$ of a cord, and $\frac{3}{8}$ of a cord=,375.

4|9,375

Ans. $23,4375

Had the multiplier been $,25, 4 cords would have cost a dollar, and in that case the point would have been placed between 2 and 3, thus, $2,34375. But as the multiplier is ten times as large as 25 cents, therefore the answer obtained by $2,5 must be ten times as large as that obtained by multiplying by $,25. This product is made ten times larger than $2,34375, by removing the point one figure to the right, thus, $23,4375, which is the same as multiplying by 10.

8. What will 476$\frac{7}{8}$ acres of land cost, at $33,33$\frac{1}{3}$ per acre?

See Table. $\frac{7}{8}$=,875.

3|476,875

Ans. $15895,83$\frac{1}{3}$.

9. Multiply 397,56 by 25.

4|397,56

Ans. 9939

10. Multiply 987586 by 83333$\frac{1}{3}$.

12|987586

Ans. 82298833333$\frac{1}{3}$.

The learner will perceive that ,083333$\frac{1}{3}$ are $\frac{1}{12}$ of one unit, and that 83333$\frac{1}{3}$ are $\frac{1}{12}$ of 1000000. The product of 987586, multiplied by $\frac{1}{12}$ of 1, is 82298,833333$\frac{1}{3}$. It is plain that the product of 987586, multiplied by $\frac{1}{12}$ of 1000000, will be a million times larger. Therefore, by removing the point six places to the right, we have the answer required.

11. Multiply 61893786,54272 by 333,333333$\frac{1}{3}$.

The multiplier is $\frac{1}{3}$ of 1000.

Ans. 20631262180,90666$\frac{2}{3}$.

12. Multiply 9437856798235 by 142857142857$\frac{1}{7}$.

The multiplier is $\frac{1}{7}$ of a billion. Divide the multiplicand by 7, and multiply the quotient by one billion, by removing the point twelve places to the right.

Ans. 1348265256890714285714285$\frac{5}{7}$.

13. Multiply 369786543,25 by 1250000.

The multiplier is $\frac{1}{8}$ of 10000000.

Ans. 462233179062500.

14. Multiply 126961272, by ,00166$\frac{2}{3}$.

It will be seen, by the Table, that the multiplier, in this example, is $\frac{1}{6}$ of ,01. Therefore, as ,01 is one hundred times *less* than 1, the point must be removed *two places to the left* of that place it would have occupied, had the multiplier been $\frac{1}{6}$ of 1. *Ans.* 211602,12.

15. Multiply 42878,625 by 625.

The multiplier is $\frac{5}{8}$ of 1000.

```
   42878,625
           5
   ---------
 8|214393,125
   ----------
Ans. 26799140,625
```

16. What cost 11$\frac{7}{8}$ units, at 3$\frac{1}{8}$ units per unit?

```
8|11,875           Or thus:  3,125 = 1/32 of 100.
      3                      4|11,875
  ------                       ------
  35,625                     8|2,96875
   1,484375                    --------
  --------                 Ans. 37,109375
Ans. 37,109375
```

17. At $,375 per yard, what will 7$\frac{5}{8}$ yards cost?

$,375 = $\frac{3}{8}$ of a dollar.

```
   7,625
       3
  ------
8|22,875
  ------
Ans. 2,859375
```

18. At 8,75 per acre, what cost $19\frac{4}{5}$ acres?

$8,75=\frac{7}{8}$ of 10.

19,8 7 8|138,6 *Ans.* 173,25	Or thus: 8|19,8 2,475 *Ans.* 173,25

$\frac{7}{8}$ of 10 are one eighth less than 10. Therefore, 19,8 multiplied by 10, and the product diminished by $\frac{1}{8}$ of 19,8, is the true answer.

19. At ,15 per pound, what cost 71 pounds and 7 ounces?

$,15=\frac{3}{20}$ of a dollar, or unit.

71,4375
3
20|214,3125
Ans. 10,715625

20. What cost 5 lb. and 5 oz. of tea, at ,75 per pound?

5 oz.$=$,3125 ,$75=\frac{3}{4}$

5,3125 3 4|15,9375 *Ans.* 3,984375	Or thus: subtract $\frac{1}{4}$ of the multiplicand from itself. 4|5,3125 1,328125 *Ans.* 3,984375

21. What will $488\frac{7}{16}$ units cost, at $5\frac{5}{8}$ units per unit?

$5\frac{5}{8}=\frac{45}{8}=\frac{90}{16}$.

488,4375 90 4|43959,3750 4|10989,84375 *Ans.* 2747,4609375	Or thus: 488,4375 5 2442,1875 9 8|21979,6875 *Ans.* 2747,4609375

22. What will 11 yards and $\frac{5\frac{1}{2}}{8}$ yards cost, at $,33\frac{1}{3}$ per yard? *Ans.* $3,8958\frac{1}{3}$.

23. What will 11 lbs. and 11 oz. cost, at ,16⅔ per pound?

$$\begin{array}{r|l} 6 & 11,6875 \\ \hline & \end{array}$$

Ans. 1,9479⅙

24. What will 11 units, $\frac{11}{16}$ of a unit cost, at ,2 per unit?

$$\begin{array}{r} 11,6875 \\ ,2 \\ \hline 2,33750 \end{array}$$

Ans. 2,33750

Or thus: $\begin{array}{r|l} 5 & 11,6875 \\ \hline & 2,3375 \end{array}$

Ans. 2,3375

25. What will 19 bushels and 11 quarts of corn cost, at $,66⅔ per bushel?

$$\begin{array}{r} 19,34375 \\ 2 \\ \hline 3|38,68750 \\ \hline 12,89583\frac{1}{3} \end{array}$$

Ans. 12,89583⅓

Or thus: $\begin{array}{r|l} 3 & 19,34375 \\ \hline & 6,44791\frac{2}{3} \\ \hline & 12,89583\frac{1}{3} \end{array}$

Ans. 12,89583⅓

At a dollar per bushel, there would be as many dollars as bushels; hence, at ⅔ of a dollar per bushel, the number of dollars will be ⅓ less than the number of bushels.

DIVISION OF DECIMALS.

1. Divide 1,25 by ,25.

$$\begin{array}{r|l|l} ,25 & 1,25 & 5, \\ & 1,25 & \\ & \overline{} & \end{array}$$

Ans. 5.

Or thus: $\begin{array}{r} 1,25 \\ 4 \\ \hline 5,00 \end{array}$

Ans. 5,00

As ,25 express ¼ of a unit, it will be contained in 1,25 as many times as there are like portions of a unit (that is, one fourths) in 1,25. And, as there are 4 fourths in a unit, multiplying by 4 gives the number of 4ths in 1,25. The same principle applies in any other case, and in relation to any other denomination of parts, as, 5ths, 6ths, 7ths, 8ths, &c.

2. Divide 1,25 by 25.

25 | 1,25 | ,05
   125   *Ans.* ,05.

Or thus: 1,25
4
Ans. ,0500

As 25 are $\frac{100}{4}$, the process of dividing by 25 may be performed, as above, by multiplying by 4, and dividing by 100; or, which is the same thing, removing the point two places to the left.

3. Divide 83796 by 125.

The divisor is $\frac{1}{8}$ of a thousand. 83796
8
Ans. 670,368

4. Divide 375875 by 333,33$\frac{1}{3}$.

The divisor is $\frac{1}{3}$ of a thousand. 375875
3
Ans. 1127,625

5. Divide 375875 by ,0033333333$\frac{1}{3}$.

The divisor is $\frac{1}{3}$ of ,01. 375875
3
Ans. 112762500

As ,003$\frac{1}{3}$ are $\frac{,01}{3}$, therefore the operation of dividing by ,003$\frac{1}{3}$ may be performed by multiplying by 3, and dividing the product by ,01; or which is the same as annexing two ciphers.

6. Divide 79625,876 by ,0016$\frac{2}{3}$.

The divisor is $\frac{1}{6}$ of ,01. 79625,876
6
Ans. 47775525,6

7. Divide 43657,875 by ,0000833333$\frac{1}{3}$.

The divisor is $\frac{1}{12}$ of ,001. *Ans.* 523894500.

8. Divide 463729, by 1428,57$\frac{1}{7}$.

The divisor is $\frac{1}{7}$ of ten thousand. *Ans.* 324,6103.

9. Divide 437,895 by 2000.

The divisor is $\frac{1}{5}$ of ten thousand. *Ans.* ,2189475.

10. Divide 4862 by ,0009$\frac{1}{11}$. *Ans.* 5348200.

11. Divide 9875 by ,000005. *Ans.* 1975000000.

12. Divide 9875 by 75.

$9875 \div 3 \times 4 = 13166\frac{2}{3} \div 100 = 131,66\frac{2}{3}$. *Ans.* 131,66$\frac{2}{3}$.

13. Divide 37694 by 375. *Ans.* 100,517$\frac{1}{3}$.

14. Divide 86975,1 by 18,75.

The divisor is $\frac{3}{16}$ of 100. The operation may be performed by multiplying by the denominator, and dividing by the numerator, and then dividing by 100, or removing the point two places to the left.

$86975,1 \times 16 \div 300 = 4638,672$, *Ans.*

15. Divide 437962 by 66,6$\frac{2}{3}$.

$437962 \div 2 \times 3 \div 100 = 6569,43$. *Ans.* 6569,43.

16. If 7 pounds cost $\frac{21}{25}$ of a dollar, what costs 1 pound?

$\frac{1}{25} = ,04 \times 21 = ,84 \div 7 = ,12$. *Ans.* ,12.

XVI. Multiplication of whole Numbers and Decimal Parts.

TO MULTIPLY BY 9, 99, 999, OR ANY NUMBER OF 9s.

Annex to the multiplicand as many ciphers as there are 9s in the multiplier, and from that product subtract the multiplicand.

1. Multiply 75 by 9.	2. Multiply 43625 by 999.
750	43625000
75	43625
Ans. 675	*Ans.* 43581375

As the multiplier, in these cases, wanted only 1 of being 10, and 1000, therefore, multiplying by these numbers would repeat the multiplicand once too many. For this reason, the multiplicand must be subtracted from the product in each case.

TO MULTIPLY BY ANY NUMBER NEARLY EQUAL TO 10, 100, 1000, ETC.

If the multiplier be less than 10, *annex one cipher to the multiplicand, and multiply the multiplicand by the excess of* 10 *over the multiplier, and subtract this product from the given multiplicand multiplied by* 10. *The same rule may be observed in numbers nearly equal to* 100, 1000, *by annexing two ciphers for* 100, *three for* 1000, *&c.*

1. Multiply 378 by 8.

378
2 the excess of 10 over 8.
756

3780 twice the multiplicand too many.
756

Ans. 3024

2. Multiply 4376 by 7.

4376
3 excess of 10 over 7.
13128

43760 three times the multiplicand too many.
13128

Ans. 30632

3. Multiply 496 by 95.

49600
2480

Ans. 47120

4. Multiply 9786 by 92.

978600
78288

Ans. 900312

5. Multiply 7368 by 75.

4|736800
184200

Ans. 552600

6. Multiply 43628 by 87,5.

8|4362800
545350

Ans. 3817450

7. Multiply 64834 by 91⅔.

12|6483400
540283,33⅓

Ans. 5943116,66⅔

8. Multiply 12934 by 666⅔.

3|12934000
4311333⅓

Ans. 8622666⅔

9. What will 19 bushels and 11 quarts of corn cost, at ,83⅓ per bushel?

6|19,34375
3,22395⅚

Ans. 16,11979⅙

10. What cost 63 yards and 15/16 of a yard, at ,97 per yard?

Ans. 62,019375.

TO MULTIPLY BY ANY NUMBER BETWEEN 10 AND 20.

Multiply the given number by 10, *and to that product add the product of the multiplicand multiplied by the unit figure.*

1. Multiply 378 by 15.

$$378\times10=3780$$
$$378\times5=1890$$
Ans. 5670

2. Multiply 4378 by 19.

43780
39402
Ans. 83182

3. Multiply 56789 by 16.

568790
341274
Ans. 910064

The same principle will enable us to perform multiplication in an abbreviated method, when the multiplier is a larger number than 20.

1. Multiply 4398 by 106.

439800
26388
Ans. 466188

2. Multiply 89734 by 1009. *Ans.* 90541606.
3. Multiply 657183 by 10008. *Ans.* 6577087464.
4. Multiply 2385749 by 108. *Ans.* 257660892.
5. Multiply 186402 by 1001. *Ans.* 186588402.

TO MULTIPLY BY 21, 31, 41, 51, 81, 91, 301, 40001, ETC.

Multiply by the figure in the column of tens in the multiplier, and place the first figure in the product under the column of tens in the multiplicand, and add it to the multiplicand.

To multiply by a number containing hundreds; multiply

by the figure in the column of hundreds, and place the first figure of the product under hundreds, and add the product to the multiplicand as before.

The same principle must be observed in larger numbers.

1\. Multiply 3783 by 21.

	3783
	7566
Ans.	79443

2\. To multiply 46783 by 31.

	46783
	140349
Ans.	1450273

3\. Multiply 56893 by 41. *Ans.* 2332613.

4\. Multiply 783 by 301.

	783
	2349
Ans.	235683

5\. Multiply 43265625 by 40001. *Ans.* 1730668265625.

6\. Multiply 983 by 8001. *Ans.* 7864983.

7\. Multiply 736952 by 90000001.
Ans. 66325680736952.

8\. Multiply 65432 by 700000001.
Ans. 45802400065432.

9\. Multiply 4375 by 60000000000000001.
Ans. 262500000000000004375.

TO MULTIPLY PARTS, AND PARTS OF PARTS, BY PARTS, AND PARTS OF PARTS.

Call that denomination whole parts, according to the nature of the question, and reduce the lower denominations to the decimal of the whole parts. Place the decimal point between the whole parts and the decimal of these parts, and multiply as in multiplication of decimals.

To reduce this product to whole numbers; take that num-

ber of parts that make a unit, according to the parts of the multiplicand, and take that number of parts that make a unit, according to the parts of the multiplier, and consider them component parts of the divisor; by this divisor divide the product obtained by multiplying parts and decimal of parts by the same, and the quotient will be the answer in units and decimal parts. In some cases, the component parts of the divisor may be multiplied together before dividing, and this product used as a divisor.

1. Multiply 3 s. by 3 s., and give the answer in U. S. money.

3 s.	6 s. make a dollar.
3 s.	6 s. " "
6|9	36
6|1,5	3 s. are $\frac{3}{6}$ of a dollar.
Ans. $,25	$\frac{3}{6}\times\frac{3}{6}=\frac{9}{36}=,25.$

2. Multiply 3 s. by 3 s., pounds taken as whole numbers.

3 s.	20 s. make one pound.
3 s.	20 s. " "
400|9	400 3 s. are $\frac{3}{20}$ of a pound.
Ans. £ ,0225=5 d. $1\frac{3}{5}$ qr.	$\frac{3}{20}\times\frac{3}{20}=\frac{9}{400}$ of a pound. $\frac{9}{400}=,0225=5$ d. $1\frac{3}{5}$ qr.

3. Multiply 6 s. by 6 s., dollars taken as units.

6 s.	6th parts.
6 s.	6th "
36|36	36 parts.
Ans. 1 dollar.	

4. Multiply 5 s. 3 d. by 5 s. 3 d., one dollar being the unit.

$$\frac{5,25}{6}\times\frac{5,25}{6}=\frac{27,5625}{36}=Ans.\ ,765625.$$

5. Multiply 4 s. 6 d. by 4 s. 6 d., calling one dollar a unit.

4,5	6	
4,5	6	4 s. 6 d.$=\frac{4\frac{1}{2}}{6}$ of a dollar.
225	36	
180		$\frac{4\frac{1}{2}}{6}\times\frac{4\frac{1}{2}}{6}=\frac{20\frac{1}{4}}{36}=$,5625, or 56¼ cts.
6|20,25		
6|3,375		

Ans. $,5625

6. Multiply 5 s. 6 d. N. Y. by 4 s. 6 d. N. E., calling one dollar a unit.

5,5 s. N. Y., or 8th parts.	8
4,5 s. N. E., or 6th parts.	6
275	48
220	
8|24,75	
6|3,09375	
Ans. ,515625	*Ans.* $,51$\frac{9}{16}$.

[In the following examples, from 7 to 33, are shown thirty different ways of working the same sum with a similar result.

7. Multiply $1,75 by $1,75.

1.	2.	3.	
1,75	4|1,75	8|2|1,75...	product produced by multiplying by ,5, which was performed by dividing by 2.
1,75	,4375		
875	,4375	,875	
1225	,4375	2,1875	
175	$3,0625	$3,0625.	product obtained by multiplying by 1,25=⅛ of 10.

Ans. $3,0625

8. Multiply 10 s. 6 d. by 10 s. 6 d. N. E., $1 being the unit.

$$\frac{10,5}{6\text{ s.}}\times\frac{10,5}{6\text{ s.}}=\frac{110,25}{36}=\$3,06\tfrac{1}{4}.\ \text{Ans.}$$

9. Multiply 10 s. 6 d. N. E. by 14 s. N. Y., one dollar being unit.

10,5 s. 6th parts..........6
14 s. 8th parts..........8

8|147,0 48

6|18,375

Ans. $3,0625

$\frac{10,5}{6}$ s. $\times \frac{14}{8}$ s. $= \frac{147}{48} = 3,06\frac{1}{4}$.

10. Multiply 8 s. 9 d. Canada currency, by 14 s. N. Y., one dollar being the unit.

8,75 s. 5th parts.......5
14 s. 8th parts.......8

3500 40
875

40|122,50

Ans. 3,0625

11. Multiply 14 s. N. Y. by 14 s. N. Y., one dollar being the unit.

14 8th parts
14 8th parts

56 64
14

8|196

8|24,5

Ans. $3,0625

$\frac{14}{8} \times \frac{14}{8} = \frac{196}{64} = 3,0625$.

12. Multiply 3,5 half parts by 7 fourth parts.

3,5 2 parts 2
7 4th parts...4

8|24,5 8

Ans. 3,0625

$\frac{3,5}{2} \times \frac{7}{4} = \frac{24,5}{8} = 3,0625$.

13. Multiply 8 s. 9 d. Canada currency, by 10 s. 6 d. N. E., one dollar being the unit.

10,5	6th parts.		6
8,75	5th	"	5
525			30
735			
840			

30|91,875

Ans. \$3,0625

14. Multiply 10 s. 6 d. N. E. by the same, one dollar being the unit.

10 s. 6 d.=1¾ units.

4|10,5....6th parts.
2,625
2,625
2,625

6|18,375

Ans. \$3,0625

15. Multiply 8,75 fifth parts by 175 one hundredth parts.

8,75	5th parts	5
175,	100th "	100
4375		500
6125		
875		

500|1531,25

Ans. 3,0625

$\frac{8,75}{5}\times\frac{175}{100}=\frac{1531,25}{500}=$ [3,0625.

16. Multiply 1,75 by $\frac{7}{4}$, one dollar being the unit.

1,75
7....4th parts.

4|12,25

Ans. \$3,0625

17. Multiply 7 fourth parts by 7 fourth parts, one dollar being the unit.

7
7

4|49

4|12,25

Ans. \$3,0625

$\frac{7}{4}\times\frac{7}{4}=\frac{49}{16}=3,0625.$

18. Multiply 10,5-6th parts by 7-4th parts.

10,5	6
7	4
4|73,5	24

6|18,375

Ans. 3,0625

19. Multiply 1,75 by 14 s. N. Y.

1,75	
14	8th parts.
700	8
175	

8|24,50

Ans. 3,0625

20. Multiply 8 s. 9 d. Canada currency, by the same, one dollar being the unit.

$$\begin{array}{rl} 4|8,75 & \text{5th parts.} \\ 2,1875 & \\ 2,1875 & \\ 2,1875 & \\ \hline 5|15,3125 & \\ \hline \end{array}$$

Ans. 3,0625

21. Multiply 168 d. N. Y. by 126 d. N. E., one dollar taken as the unit.

$$\begin{array}{rlr} 168 & \text{96th pts.} & 8\times12 \\ 126 & 72\ \text{“} & 8\times9 \\ \hline 1008 & & \\ 336 & & \\ 168 & & \\ \hline 8|21168 & & \\ \hline 9|2646 & & \\ \hline 8|294 & & \\ \hline 12|36,75 & & \\ \hline \end{array}$$

Ans. 3,0625

$\frac{168}{96}\times\frac{126}{72}=\frac{21168}{6912}=3,0625.$

22. Multiply 3,5 half parts by 3,5 half parts.

$$\begin{array}{rl} 3,5 & 2\ \text{parts.} \\ 3,5 & 2\ \text{parts.} \\ \hline 175 & 4 \\ 105 & \\ \hline 4|12,25 & \\ \hline \end{array}$$

Ans. 3,0625

$\frac{3,5}{2}\times\frac{3,5}{2}=\frac{12,25}{4}=3,0625.$

23. Multiply 175 cents by 126 d. N. E., one dollar taken as the unit.

$$\begin{array}{rl} 8|175 & \text{100th pts.} \\ 126 & 72\ \text{“} \\ \hline 21875 & \\ 175 & \\ \hline 8|220,50 & \\ \hline 9|27,5625 & \\ \hline \end{array}$$

Ans. 3,0625

$\frac{175}{100}\times\frac{126}{72}=\frac{22050}{7200}=3,0625.$

24. Multiply 672 qrs. N. Y. by 504 qrs. N. E., one dollar taken as the unit.

672 384th parts.
504 288th parts.

$\frac{672}{384}\times\frac{504}{288}=\frac{338688}{110592}=\$3,0625$, *Ans.*

25. Multiply $7\frac{7}{8}$ s. English money, by $8\frac{3}{4}$ s. Canada, one dollar taken as the unit. *Ans.* \$3,0625.

26. Multiply 10 s. 6 d. N. E. by $7\frac{7}{8}$ s. English money, one dollar taken as the unit. *Ans.* \$3,0625.

27. Multiply 8,16$\frac{2}{3}$ s. Georgia currency, by the same, one dollar being the unit. *Ans.* \$3,0625.

28. Multiply 8 s. 9 d. Canada currency, by 13$\frac{1}{8}$ s. Pennsylvania currency, one dollar being taken as the unit. *Ans.* \$3,0625.

29. Multiply 13$\frac{1}{8}$ s. Pennsylvania currency, by 42 three penny pieces, N. E., one dollar being the unit. *Ans.* \$3,0625.

30. Multiply 13$\frac{1}{8}$ s. Pennsylvania, by 8$\frac{1}{6}$ s. Georgia currency, one dollar being the unit. *Ans.* \$3,0625.

31. Multiply 8$\frac{1}{6}$ s. Georgia currency, by 7$\frac{7}{8}$ s. English money, one dollar being the unit. *Ans.* \$3,0625.

32. Multiply 52,5 three penny pieces, Pennsylvania currency, by 31,5 three penny pieces English money, one dollar taken as the unit. *Ans.* \$3,0625.

33. Multiply 7 twenty-five cent pieces by 52,5 three penny pieces, Pennsylvania currency, one dollar being the unit. *Ans.* \$3,0625.

The scholar should make himself familiar with the principle on which the foregoing questions are performed, and be able, not only to work the sums given above, but also to vary the process himself, and thus add other forms to the thirty different ways in which the same sum has here been done.

It is not necessary to call the answer dollars; it will, with equal propriety, represent units and parts of units of any kind or denomination, as dollars and parts of dollars.

34. At 3 bushels and 7 quarts of grain per yard for broadcloth, how much grain must be given in exchange for 11$\frac{1}{16}$ yards of cloth? or, Multiply 11$\frac{1}{16}$ yards of broadcloth by 3 bushels and 7 quarts of grain.

Ans. 37,619140625, or 37 bu. 2 pk. 3 qts. 6,5 gills.

35. What will 37 pounds and 3 ounces of tea cost, at 1 bushel and 1 peck of oats per pound?

Ans. 46,484375 bushels of oats=46 bu. 1 pk. 7 qts. 1 pt.

36. Multiply 7 s. 7 d. N. Y. currency, by 5 s. 5 d. N. E. one dollar being the unit.

7 s. 7 d.	5 s. 5 d.	91..96th parts.
12	12	65..72 "
91 d. N.Y.	65 d. N. E.	455
		546
		8|5915
		8|739,375
		9|92,421875
		12|10,269097$\frac{2}{9}$

Ans. \$,855758$\frac{11}{108}$.

$\frac{91}{96}\times\frac{65}{72}=\frac{5915}{6912}=$,855758$\frac{11}{108}$.

37. Multiply 11 s. 11 d. Canada currency, by 7 s. 7 d. N. E., and give the answer in federal money.
Ans. \$3,0122$\frac{37}{54}$.

38. What will 11$\frac{1}{12}$ 8th parts of a unit cost, at 7$\frac{7}{12}$ 6th parts of a dollar, per unit? *Ans.* 1,882$\frac{577}{864}$.

39. Multiply 11 s. 9 d. N. E. by 7 s. 3 d. N. Y., one dollar being the unit. *Ans.* \$1,77473958$\frac{1}{3}$.

40. What will 18$\frac{3}{8}$ 7th parts of a unit cost, at 9$\frac{4}{5}$ 9th parts of a dollar per unit? *Ans.* \$2,8875.

41. What will 7$\frac{3}{4}$ 8th parts of a unit cost, at 8$\frac{1}{4}$ 5th parts of a bushel of wheat per unit?
Ans. 1,5984375 bushel and part=1 bu. 19 qts. 1$\frac{1}{5}$ gill.

42. What will 93$\frac{13}{16}$ yards of cloth cost, at 3 pounds and 2 ounces of tea per yard?
Ans. 293,1640625 lbs.=293 lbs. 2 oz. 10 dr.

43. What will 7 cords and 7$\frac{1}{2}$ cord feet of wood cost, at 9 bushels and 12 quarts of apples per cord?
Ans. 74,4140625 bushels=74 bu. 1 pk. 5 qts. 2 gills.

44. What will 37$\frac{7}{8}$ acres of land cost, at 21$\frac{7}{8}$ cords of wood per acre?
Ans. 828,515625 cords=828 cords, 4$\frac{1}{8}$ cord feet.

45. Multiply 2 s. 6 d. by 2 s. 6 d., one pound taken as the unit.

```
        2,5  20th parts......20
        2,5  20th parts......20
        ---                  ---
        125                  400
        50
        ---
    400|6,25
        ------
      ,015625 of a £.
            20
      --------
      ,312500 of a shilling.
            12
      --------
        3,7500 pence and decimal of a penny.
           4
        ------
    3,00 farthings.                 Ans. 3 d. 3 qrs.
```

$\frac{2,5}{20}\times\frac{2,5}{20}=\frac{6,25}{400}=,015625$ of a £ $=$ 3 d. 3 d. 3 qrs.

46. Multiply 2 s. 6 d. by the same, one dollar taken as the unit. *Ans.* ,1736$\frac{1}{9}$.

47. What will 8$\frac{7}{20}$ 11th parts of an acre of land cost, at 76 9th parts of a dollar per acre? *Ans.* \$6,41$\frac{1}{99}$.

48. Multiply 11 s. 11 d. N. Y. currency, by 7 s. 7 d. N. E. currency, and that product multiply by 5 s. 5 d. Canada currency, and give the answer in federal money.
Ans. \$2,039$\frac{5773}{10368}$.

49. Multiply 2 s. 6 d. by the same, a shilling taken as the unit. *Ans.* 6,25 s. $=$ 6 s. 3d.

50. What cost 7$\frac{4}{5}$ eighth parts of a ton of hay, at 75 6th parts of a dollar per ton? *Ans.* \$12,1875.

51. What cost 71$\frac{9}{20}$ 8th parts of a cord of wood, at 47$\frac{3}{5}$ 12$\frac{1}{2}$ parts of a yard of cloth per cord? *Ans.* 34,0102 yds.

$\frac{71,45}{8}\times\frac{47,6}{12,5}=\frac{3401,02}{100,}=34,0102$, *Ans.*

52. Multiply 1$\frac{4}{7}$ by 1$\frac{2}{7}$. *Ans.* 2,02$\frac{2}{49}$.

53. If 1 thing cost $\frac{5}{6}$ of a dollar, what cost $\frac{4}{5}$ of it?
Ans. \$,66$\frac{2}{3}$ $=\frac{2}{3}$.

54. Multiply £11 11 s. 11 d. by £11 11 s. 11 d.

```
    £11 11 s. 11 d.
       20
      ----
      231
       12
     -----
     2783 d.....240th parts
     2783 d.....240th parts
     -----
     8349
    22264
   19481
   5566
 ---------
60|7745089..48/60 parts of a farthing left here.
  ---------
 4|129084....960th parts, or farthings.
  --------
 12|32271..240 parts, or pence.   3 d. left here.
   -------
  20|2689..20th parts, or shillings.   9 s. left here.
    ------
     134
```

Ans. £134 9 s. 3 d. 0 qr. and $\frac{48}{60}$ of a farthing.

55. Multiply £3 6 s. 8 d. by £2 5 s. 7 d.
Ans. £7 11 s. 11 d. $1\frac{1}{3}$ qr.

56. Multiply 8640 qrs. by 3000 cents, one pound taken for a unit. *Ans.* £81.

57. Multiply 8640 farthings by 3000 cents, one shilling taken as the unit. *Ans.* 32400 s.

58. Multiply 8640 qrs. by 3000 cents, one dollar taken as the unit. *Ans.* 900.

59. Multiply £19 19 s. 11 d. 3 qrs. by the same.
Ans. £399 19 s. 2 d. $0\frac{.0625}{60}$ qr.

EXAMPLES FOR PRACTICE.

1. A merchant sold six parcels of cloth; the first contained 421$\frac{1}{20}$ yards; the second, 97$\frac{7}{1000}$; the third, 900$\frac{7}{100}$; the fourth, 67$\frac{4}{10000000}$; the fifth, 27$\frac{18}{20}$; the sixth, one fourth as many as the first five parcels: how much cloth did he sell, and how much did the whole cost, at \$1,66$\frac{2}{3}$ per yard? *Ans.* 1891,346255 yards.
\$3152,243758$\frac{1}{3}$, whole cost.

2. A merchant bought 4 chests of tea; the first contained 76 pounds, 11 oz.; the second, 59 lb. 13 oz.; the third, 64 lb. 15 oz.; and the fourth, half as much as the first. How much did he buy, and what did it cost, at 5 s. per lb.?
Ans. Amt. 302,15625=302 lbs. 2$\frac{1}{2}$ oz. \$251,796875.

3. A horse dealer made in one trade, $\frac{3}{8}$ of \$100; in another, $\frac{7}{8}$ of \$10; in another, $\frac{15}{16}$ of \$10; in another, $\frac{11}{20}$ of \$100; but in another, he lost $\frac{3}{20}$ of \$1000. Did he gain or lose, and how much? *Ans.* He lost \$39,375.

4. A farmer sold to a merchant 9 cords and 6 cord feet of wood, at \$1,25 per cord; 71 bushels and 16 quarts of wheat, at \$2,5 per bushel; 65 bushels and 12 quarts, at \$,5 per bushel. He took in exchange a web of sheeting, of 37 yds. at \$,16$\frac{2}{3}$ per yd.; a web of shirting, of 31$\frac{5}{8}$ yds. at \$,125 per yd.; a pair of boots, at \$3$\frac{3}{4}$; a hat, at \$4,5; and the rest in money. How much money did he receive?
Ans. \$205,255208$\frac{1}{3}$.

5. Suppose a man wills to seven heirs his property: to the first, $\frac{7}{8}$ of \$1000; to the second, $\frac{9}{16}$ of \$1000; to the third, $\frac{5}{8}$ of \$1000; to the fourth, $\frac{15}{16}$ of \$1000; to the fifth, $\frac{16}{20}$ of \$1000; to the sixth, $\frac{3}{8}$ of \$10000; and to the seventh, as much as to the other six: how much did he will to them all? *Ans.* \$15100.

6. If 1 pound of wool make 60 knots of yarn, how many skeins of ten knots each will 4 lbs. 6 oz. make?

$$\begin{array}{r} 4,375 \\ 60 \\ \hline 262,500 \text{ knots.} \end{array}$$

Ans. 26,25 skeins.

7. If 1 pound make $49\frac{3}{8}$ knots, how many knots will 12 pounds and 8 oz. make?

$$8|49,375$$

Ans. $617,1875 = 617\frac{3}{16}$.

8. A merchant bought one chest of tea that contained 81 pounds and 7 ounces, at \$,75 per pound; he bought another chest of 79 pounds, 11 ounces, at \$,625 per pound; one keg of tobacco of 98 pounds and 15 ounces, at \$,$16\frac{2}{3}$ per pound; another of 77 pounds, 9 ounces, at \$,25 per pound; 3 boxes of raisins, of 31 pounds each—one box at \$,$08\frac{1}{3}$ per pound, one at \$,125 per pound, the other at \$,$16\frac{2}{3}$ per pound; 10 webs of sheeting, of $34\frac{3}{8}$ yards each, at \$,$11\frac{1}{9}$ per yard; and $80\frac{1}{8}$ yards of broadcloth, at \$$3,33\frac{1}{3}$ per yard; what was the amount of the whole?

Ans. \$$463,66579861\frac{1}{9}$.

9. Multiply $\frac{7}{8}$ of a shilling by $\frac{3}{4}$ of a penny, one pound taken as the unit. *Ans.* ,13125 of a farthing, or £ ,00013671875.

If one dollar had been taken as the unit, then the answer would have been, \$,$001519097\frac{2}{9}$.

Had a shilling been taken for a unit, the answer would have been, ,0546875 of a s.

Had a penny been the unit, the answer would have been, 7,875 d.$=7\frac{7}{8}$ d.

Had a farthing been the unit, the answer would have been, 126 qrs.

10. What will $\frac{3}{5}$ of $\frac{7}{8}$ of a thing cost, at 2,5 per thing?

Ans. 1,3125.

11. What will $\frac{4,5}{5}$ of $\frac{15}{16}$ of a pound cost, at \$1,25 per pound? *Ans.* \$$1,0546\frac{7}{8}$.

12. What will $\frac{2,5}{5}$ of $\frac{3}{16}$ of an acre of land cost, at \$$166,66\frac{2}{3}$ per acre? *Ans.* \$15,625.

13. At 1 s. 3 d. per bushel, how many bushels of potatoes must be given in exchange for 19 s. 9 d.?

Ans. 15 bu. 3 pecks, 1 qt. and 4,8 gills.

14. How much tea, at 2 s. 3 d. per pound, must be given in exchange for $19\frac{5}{8}$ yds. of cloth, at 6 s. 9 d. per yard?

Ans. 58 pounds, 14 oz.$=58,875$.

15. How many bushels of apples, at 1 s. 6 d. per bushel, must be given in exchange for 3 cords and 7 cord feet of wood, at 11 s. per cord? *Ans.* 28 bu. and $\frac{5}{12}$ of a bushel, or $13\frac{1}{3}$ qts.=$28,41\frac{2}{3}$.

16. At 1 cord and 2 cord feet of wood per yard for broadcloth, how much wood must be given in exchange for 11 yds. and $\frac{11}{16}$ of a yard? *Ans.* 14,609375=14 cords, 4 cord feet, and 14 cubic feet, or 14 cords and 4,875 cord feet.

17. At 2 bushels and 12 quarts of grain per unit, how much grain will be required to buy $79\frac{13}{16}$ units? *Ans.* 189,5546875=189 bushels, $17\frac{3}{4}$ qts.

18. At $12\frac{1}{2}$ yards of cloth per cord, how much cloth will be required to purchase 17 cords and $7\frac{1}{2}$ cord feet? *Ans.* 224,21875 yds.

19. Multiply $11\frac{5}{16}$ pounds by 3 bushels and 7 quarts, or $11\frac{5}{16}$ units by $3\frac{7}{32}$ units. *Ans.* 24,74609375.

XVII. Interest.

Interest is a premium paid for the use of money. *Legal* interest is that which is allowed by law. The money paid for the use of $100, or $1,00, is called the rate *per cent;* that is, *by the hundred.* The sum let, is called the *principal.* The principal and interest added, produce the *amount.* In New England, the *legal* rate *per cent* is 6. In New York, 7 per cent is the legal rate, and in Louisiana, 8 per cent.

TO COMPUTE INTEREST AT ANY RATE PER CENT.

RULE.—*Find the time in years and decimal parts of a year; that is, reduce the months and days to the decimal of a year, and place it at the right hand of the year, or years, if there be any. Multiply this time by the rate per*

cent, (which is the interest on one dollar for one year,) *and this product multiply by the given sum.*

Note.—*In computing interest in this manner*, 30 *days are reckoned one month, and* 12 *months, or* 360 *days, one year.*

1. What will be the interest on $75, for one year and 6 months, at 6 per cent?

1 yr. 6 m.=1,5 time.
,06 interest on 1 dollar for 1 year.

,090 interest on 1 dollar for 1 yr. and 6 m.
75

Ans. $6,750 interest on $75 for 1 year and 6 m.

2. What will be the interest on $88 for one year and 9 months, at 5 per cent?

1 yr. 9 m. = 1,75 time.
,05 interest on $1 for 1 yr. at 5 per cent.

,0875 interest on $1 for 1 yr. and 9 m.
88

7000
7000

Ans. $7,7000 interest on $88 for 1 yr. 9 months.

3. What will be the interest on $44,58 for 2 years and 3 months, at 7 per cent?

2 yrs. 3 m.=2,25 time.
,07 interest on $1 for 1 year.

,1575 interest on $1 for 2 years, 3 months.
44,58

,012600 interest of $,08 for 2 years, 3 m.
,07875 interest of $,5 for 2 years, 3 m.
,6300 interest of $4 for 2 years, 3 mo.
6,300 interest of $40, for 2 years, 3 m.

Ans. $7,021350 interest on $44,58 for 2 years, 3 m.

4. What will be the interest on $85,06 for 3 years, 1 month, and 15 days, at $9\frac{3}{4}$ per cent?

3 yrs. 1 m. 15 d.=3,125 time.
,0975 interest on $1 for 1 year at $9\frac{3}{4}$ [per cent.

15625
21875
28125

,3046875 interest on $1 for 3 yrs. 1 m. [15 days, at $9\frac{3}{4}$ per cent.
85,06

18281250
15234375
24375000

Ans. 25,916718750 interest on $85,06 for 3 years, [1 m. 15 d. at $9\frac{3}{4}$ per cent.

TO REDUCE MONTHS AND DAYS TO THE DECIMAL OF A YEAR.

Reduce the months to days, and to this sum add the given days. Divide the whole number of days by 360, *because* 360 *days, in computing interest, are reckoned* 1 *year, and the quotient will be the decimal of a year. The component parts of* 360 *may be employed, which are* 10, 4, *and* 9.

5. What will be the interest of $7500 for $96\frac{3}{4}$ days, at 6 per cent?

4|9'6,75
9|2,41875
,26875 time.
,06

,0161250
7500

80625000
1128750

Ans. $120,9375000

6. What will be the interest on $375,875 for 3 years, 11 months, and 19 days, at 5 per cent?

3 yrs. 11 m. 19 d.=3,969⅙ time.
,05

,19845⅚ interest on $1 for 3 years, [11 m. 19 d. at 5 per cent.

,19845⅚×375,875=*Ans.* 74,594481944⁄6.

In this example, the whole answer is given, and it would be well for the learner to do all his sums in this way. It is plain, however, that in business no account is taken of a denomination lower than mills. Had we called the time 3,96911, the answer would have been, $74,5944640625, which is sufficiently accurate for all practical purposes.

7. What is the interest of $496,256 for 9 months, 23 days, at 11 per cent? *Ans.* $44,4286966⁄8.

8. What is the interest of $396,45 for 10 years, 3 m. and 12 days, at 6 per cent? what, at 9 per cent?

Ans. at 6 per cent, $244,61965.
at 9 per cent, $366,929475.

9. What is the interest of $378,35 for one year, 3 m. and 21 d. at 9 per cent? *Ans.* $44,5507125.

ONE METHOD OF PROVING SUMS IN INTEREST.

Multiply the principal by the rate rate per cent, which gives the interest for one year. This product multiply by the time.

10. What is the interest of $142,857⅐ for 4 years, 11 m. and 21 d. at 12½ per cent?

4 yrs. 11 m. 21 d.=4,975.
8|4,975×,125 (rate per cent=⅛.)

7|,621875 interest on $1 for 4 yrs. 11 m. 21 d.

Ans. 88,839$\frac{2}{7}$

$142,857⅐=⅐ of $1000.

11. What is the interest of \$7,20 for 79 days, at 6 per cent? *Ans.* \$,0948

12. What is the interest of \$120 for 49 days, at 6 per cent? *Ans.* \$,98.

13. What is the interest of \$83,33⅓ for 3 years, 9 m. and 9 days, at 6¼ per cent?

3 yrs. 9 m. 9 d.=3,775×,0625 (rate per cent=$\frac{1}{16}$.)

4|3,775 time.

4|,94375

12|,2359375 int. of \$1 for 3 yrs. 9 m. 9 d.

Ans. \$19,6614$\frac{7}{12}$

\$83,33⅓=$\frac{1}{12}$ of 1000.

14. What is the interest of \$87,5 for 19 years, 4½ days, at 8⅓ per cent?

19 yrs. 4,5 d.=19,0125, time. ,08⅓=$\frac{1}{12}$ rate per cent.

\$87,5=$\frac{7}{8}$ of 100. 12|19,0125

1,584375 interest of \$1 for the [whole time.

7

8|11,090625

Ans. \$138,6328125

STANDARDS FOR COMPUTING INTEREST,

AT DIFFERENT RATES PER CENT.

A STANDARD *is the number of days in which money, at simple interest, and at any given rate, will double.*

TO FIND THE STANDARD FOR ANY RATE PER CENT.

At one per cent, any sum of money will double in 100 *years, or* 36000 *days. At* 2 *per cent, any sum of money will double in half this time, or* 50 years=18000 days

Therefore, divide 36000 days by the number representing the rate per cent, and the quotient will be the standard for that rate.

36000 are the standard for 1 per cent.

$36000 \div 2 = 18000$	2	"
$36000 \div 3 = 12000$	3	"
$36000 \div 4 = 9000$	4	"
$36000 \div 5 = 7200$	5	"
$36000 \div 6 = 6000$	6	"
$36000 \div 7 = 5142\frac{6}{7}$	7	"
$36000 \div 8 = 4500$	8	"
$36000 \div 9 = 4000$	9	"
$36000 \div 11 = 3272\frac{8}{11}$	11	"
$36000 \div 12 = 3000$	12	"

This Table may be extended farther, by continuing the division. If a standard is needed, in which a fraction occurs, divide 36000 by the given rate, whatever it may be, and the quotient will be the standard.

TO COMPUTE INTEREST BY THE STANDARDS.

Rule.—*Divide the given sum by the standard for the required rate, and the quotient will be the interest for one day. Multiply the interest for one day by the number of days, and the product will be the answer.*

Proof.—*Divide the time (reduced to days) by the standard, and the quotient will be the interest of one dollar for the given time. Multiply the interest of one dollar by the number of dollars, and the product will be the answer.*

Note.—A more full explanation of the 6 per cent standard will hereafter be given, it being our lawful rate; but all others, being of the same nature, may be worked in the same manner.

1. What is the interest of $67,58 for 3 years, 8 months, and 10 days, at 6 per cent?

```
                     6000|67,58              Proof. 6000|1330
                         -----                       -----
                         ,011263⅓                     ,221⅔
3 yrs. 8 m. 10 d.  =      1330 days.
                         -------                     67,58
                          337890                      ,221⅔
                         33789                       -------
                        11263443⅓                     6758
                        ---------                    13516
               Ans. 14,980233⅓                      13516
                                                      2252⅔
                                                      2252⅔
                                                    ---------
                                                    14,98023⅓
```

2. What is the interest of $366,42 for 4 months and 5 days, at 6 per cent? *Ans.* $7,63375.

3. What is the interest of $420 for 2 years, 9 months, and 10 days, at 6 per cent? *Ans.* $70.

The scholar should be careful, and *prove* his work.

4. What is the interest of $325 for 5 m. at 6 per cent? *Ans.* $8,125.

5. What is the interest of $7,5 for 1 year, 7 months, and 29 d. at 6 per cent? $,74875.

6. What is the interest of $666,666 for 3 yrs. 5 months, and 15 days, at 6 per cent? *Ans.* $138,333195.

7. What is the interest of $420 for 2 years, 3 months, and 1 day, at 6 per cent? *Ans.* 56,77.

When no rate per cent is mentioned, 6 per cent is understood.

8 What is the interest of $150 for 75 days, at 6 per cent? *Ans.* $1,875.

9 What is the interest of $1500 for 40 days, at 6 per cent? *Ans.* $10.

10. What is the interest of $,5 for 2 years, 9 months, and 10 days? *Ans.* $,08⅓.

11. What is the interest of $1000 for ½ of a day?
Ans. $,08⅓

12. What is the interest of $120000 for 7½ days? *Ans.* $150.

13. What is the interest of $36,36 for 9 days? *Ans.* $,05454.

14. What is the interest of $89,71 for 3 months and 11 days? *Ans.* $1,510118⅓.

15. What is the interest of $240 for 79 days, at 3 per cent? *Ans.* $1,58.

16. What is the interest of $150 for 11 days, at 3 per cent? *Ans.* $,1375.

17. What is the interest of $48,48 for 48 days, at 3 per cent? *Ans.* $,19392.

18. What is the interest of $49,5 for 89 days, at 8 per cent? *Ans.* $,979.

19. What is the interest of $22,5 for 187 days, at 8 per cent? *Ans.* $,935.

20. What is the interest of $11,25 for 270 days, at 8 per cent? *Ans.* $,675.

21. What is the interest of $56,25 for 750 days, at 8 per cent? *Ans.* $9,375.

22. What is the interest of $750 for 5625 days, at 8 per cent? *Ans.* $937,50.

23. What is the interest of $28,125 for 71 days, at 8 per cent? *Ans.* $,44375.

24. What is the interest of $703,125 for 77 days, at 8 per cent? *Ans.* $12,03125.

25. What is the interest of $140,625 for 473 days, at 8 per cent? *Ans.* $14,78125.

26. What is the interest of $6,25 for 135 days, at 8 per cent? *Ans.* $,1875.

27. What is the interest of $13,5 for 11 days, at 8 per cent? *Ans.* $,033.

28. What is the interest of $31,5 for 99 days, at 8 per cent? *Ans.* $,693.

ANOTHER METHOD OF COMPUTING INTEREST BY THE STANDARDS.

If we call 6000 *days* \$6000, *this sum of money will gain* 1 *dollar every day.* \$3000 *will gain a dollar in* 2 *days. Hence we have the following*

STANDARD.

12	72000
11	66000
9	54000
8	48000
7	42000
6	36000
4	24000
3	18000
1	6000
2	3000
3	2000
4	1500
5	1200
6	1000
8	750
12	500
24	250
25	240

In computing interest from this standard, we reason from that sum of money which gains 1 dollar per day, i. e. \$6000.

1. What is the interest of \$60 for 150 d. at 6 per cent?

100 | 150

Ans. 1,5

\$6000 gain \$1 per day; in 150 days, they would gain \$150. But \$60 are 100 times less than 6000; therefore \$60 would gain, in 150 days, 100 times less than \$6000 gain in that time. If then, the days (150) are divided by 100, the quotient will be the answer.

TO COMPUTE INTEREST ON THIS STANDARD BELOW 6000.

Divide the number of days by that number which shows how many times less the given sum is, than 6000, and the quotient will be the answer.

2. What is the interest of \$300 for 120 days?

20 | 120

Ans. \$6

3. What is the interest of \$120 for 300 days?

300÷50 = *Ans.* \$6.

4. What is the interest of \$100 for 75 days?

75÷60 = *Ans.* \$1,25.

5. What is the interest of \$50 for 20 days?

20÷120 = *Ans.* \$,16$\frac{2}{3}$.

6. What is the interest of \$20 for 50 days?

50÷300 = Ans. \$,16$\frac{2}{3}$

7. What is the interest of \$7,5 for 60 days?
$60 \div 800 =$ *Ans.* \$,075.

8. What is the interest of \$60 for 7,5 days?
$7{,}5 \div 100 =$ *Ans.* \$,075.

From these examples, the learner will perceive that *money* may be reckoned *time*, and *time money*, according to the nature of the case.

The following sums are wrought by *multiplying* the days by $12\frac{1}{2}$, the number which shows how many times *larger* the given sum (75000) is than 6000.

9. What is the interest of \$75000 for 13 days, 12 h.= 13,5 d.? *Ans.* \$168,75.

10. What is the interest of \$75000 for 11 days, 3 h. = 11,125 days?
Ans. \$139,0625.

11 What is the interest of \$75000 for 7 days, 6 h. = 7,25 days? *Ans.* \$90,625.

12. What is the interest of \$75000 for 1 day, 21 h. = 1,875 d.? *Ans.* \$23,4375.

13. What is the interest of \$75000 for 9 d. 1,5 h. = 9,0625 days?
Ans. \$113,28125.

14. What is the interest of \$75000 for 17 d. 4,5 h. = 17,1875 days?
Ans. \$214,84375.

15. What is the interest of \$75000 for 21 days, 7,5 h.= 21,3125 days?
Ans. \$266,40625.

16. What is the interest of \$75000 for 8 days, 16,5 h.= 8,6875 days?
Ans. \$108,59375.

17. What is the interest of \$75000 for 90 days?
Ans. \$1125.

The time and interest of the 17th sum are equal to the time and interest of the eight preceding sums.

\$6000 draw, in 1 day, 1 dollar. The same sum, in 90 days, would draw \$90. But \$75000 are a greater number than \$6000 by $12\frac{1}{2}$. Therefore, \$75000 will draw, in one day, $\$12\frac{1}{2}$, and in 90 days would draw, $90 \times 12\frac{1}{2} = \$1125$. Hence,

TO COMPUTE INTEREST ON THIS STANDARD ABOVE 6000, we have the following rule.

Rule.—*Multiply the number of days by that number which shows how many times larger the given sum is than 6000, and the product will be the answer.*

By using the numbers at the left of the perpendicular line, and below the unit figure, as *divisors*, interest may be computed on any sum of money, however large or small. Annex as many ciphers to the number of days, when the sum of money is larger than 6000, as the number of figures, in the given sum, exceed those at the right hand of the divisor. Thus: What is the interest of $1000000 for 30 days? In this sum the divisor is 6. At the right hand of 6, in the Standard, we find 1000. But 1000000 contain three places more than 1000; therefore three ciphers must be added to the 30 days. This amount divide by 6, and the quotient will be the answer.

30000÷6=5000. *Ans.* $5000.

When the sum of money, or the number of days, does not come directly upon the Standard, divide the sum of money, or the days, into convenient parts that come upon the Standard. Cast the interest upon these parts, and add the several interests, and the amount will be the answer. Thus:—

What is the interest of $9675,75 for 120 days?

9675,75=6000+3000+600+75+,75.

The interest of $6000 is	$120,
The interest of $3000 is	$60,
The interest of $600 is	$12,
The interest of $75 is	$1,50
The interest of $,75 is	$,015
The interest of $9675,75 is	$193,515

It will be seen, also, that in calling days dollars, and dollars days, the sum may be easily proved. Thus, calling 120 days $120, and $9675,75, 9675,75 days, and casting the interest on $120, the answer will be, $193,515.

50|9675,75

Ans. $193,515

TO COMPUTE INTEREST AT ANY RATE PER CENT ON THE SIX PER CENT STANDARD.

RULE.—*Multiply the days by the rate per cent, and divide that product by* 6*; and then perform the operation as at six per cent.* Or,

Multiply the given sum by the rate per cent, and divide by 6*; then perform the operation as at six per cent.*

1. What is the interest of $50 for 96 days, at 9 per cent?

```
          96
           9
         ———
       6|864
         ———
   12'0|14'4
       ————
Ans. $1,2
```

At 6 per cent, $50, for 96 days, would draw $,8. But $50, at 6 per cent, would require 144 days in which to draw $1,2, the interest of $50 for 96 days, at 9 per cent. There is the same relation between the 96 days and 144 days, as there is between $,8 and $1,2, and the same between $,8 and $1,2, as exists between 6 per cent and 9 per cent. It is plain, therefore, that whenever any rate per cent, other than 6, is given, it is only necessary to make the days *correspond* with the rate per cent, and then proceed as at 6 per cent.

2. What is the interest of $1,2 for 49 days, at 7⅘ per cent?

```
             49
            7,8
            ———
            392
           343
           ————
        6|382,2
          —————
     5000|63,7   time at 7⅘ per cent.
          —————
Ans. $,01274
```

3. What is the interest of \$496,256 for 9 months and 23 days, at 11 per cent? *Ans.* \$44,4286968⅝.

4. What is the interest of \$25,25 for 81 days, at 4½ per cent? *Ans.* \$,25565625.

5. What is the interest of \$2,4 for 99⅝ days, at 8¾ per cent? *Ans.* \$,05811458⅓.

It is evident, that to compute interest at 6 per cent, 30 days are reckoned a month. Therefore, to make the time correspond with the rate per month, divide the days of the month by 6, and the quotient will be the time for one month at 1 per cent.

Multiply any given rate per cent by 5, the number of days in one month at 1 per cent, and the product will be the number of days that must be reckoned a month at such a rate per cent.

To ascertain the time in odd days, multiply them by the rate per cent, divide by 6, and add the quotient to the months reduced to days. Proceed, then, as in 6 per cent.

1. What is the interest of \$736,15 for 11 m. 21 d. at 7 per cent?

```
   7   rate.                        21   odd days.
   5   d. =1 m. at 1 per cent.       7   rate.
  ---                              ---
  35   d.=1 m. at 7    "         6|147
  11   m. in the given sum.        ---
  ---                             24,5   d. at 7 per
 385   days in 11 m. at 7 per cent.              [cent.
  24,5
 -----
 409,5   days in 11 m. and 21 d. at 7 per cent.
```

Ans. \$50,2422375.

Prove the same, calling days dollars, and dollars days.

The student should perform this sum in this manner, and obtain the answer again.

2. What is the interest of \$666,666 for 2 years, 11 m. and 10 days, at 8⅝ per cent? *Ans.* \$164,39798375.

A STANDARD *being the time in which money doubles, it may be used when the time is expressed in months and years.*

as well as in days. When the time is expressed in years and decimal parts of a year, multiply the given sum of money by the given years and parts of a year, and divide by the number of years in which money doubles. When the time is expressed in months and decimal parts of a month, multiply the given sum of money by the given number of months and decimal parts of a month, and divide by the number of months in which money doubles.

To find the number of years in which money doubles, at any rate per cent, divide \$100 *by the number of dollars that* \$100 *will gain in one year, at any required rate.*

STANDARDS.

Rates per cent.	Years.	Months.	Days.
1	100	1200	36000
2	50	600	18000
3	$33\frac{1}{3}$	400	12000
4	25	300	9000
5	20	240	7500
6	$16\frac{2}{3}$	200	6000
7	$14\frac{2}{7}$	$171\frac{3}{7}$	$5142\frac{6}{7}$
8	$12\frac{1}{2}$	150	4500
9	$11\frac{1}{9}$	$133\frac{1}{3}$	4000
10	10	120	3600
11	$9\frac{1}{11}$	$109\frac{1}{11}$	$3272\frac{8}{11}$
12	$8\frac{1}{3}$	100	3000

1. What is the interest of \$478,95 for 1 year, 7 months, at 4 per cent?

By years. 478,95
$1\frac{7}{12}$

478,95
279,3875

Ans. 758,3375÷25=\$30,3335

By months. 478,95×19÷300=*Ans.* \$30,3335.

Note.—By performing division by multiplication, the examples under this rule may be performed in a very short method.

2. What is the interest of \$837,96 for 7 years and 9 months, at 7 per cent?

By years. $\$837,96\times7,75\div14\frac{2}{7}=$ *Ans.* \$454,5933.
By months. $\$837,96\times93\div171\frac{3}{7}=$ *Ans.* \$454,5933.

3. What is the interest of \$49,786 for 4 years and 11 m. at 8 per cent?

By years. $\$49,786\times4\frac{11}{12}\div12\frac{1}{2}=$ *Ans.* \$19,58249$\frac{1}{3}$.
By months. $\$49,786\times59\div150=$ *Ans.* \$19,58249$\frac{1}{3}$.

4. What is the interest of \$91,739 for 4$\frac{1}{2}$ months, at 5 per cent?

By years. $\$91,739\times,375\div20=$ *Ans.* \$1,72010625.
By months. $\$91,739\times4\frac{1}{2}\div240=$ *Ans.* \$1,72010625.

5. What is the interest of \$632,98 for 3 months, at 11 per cent? *Ans.* \$17,40695.

6. What is the interest of \$768,437 for 1$\frac{1}{2}$ month, at 9 per cent? *Ans.* \$8,64491625.

7. What is the interest of \$478,13 for 4 months, at 4 per cent? *Ans.* \$6,3750$\frac{2}{3}$.

8. What is the interest of \$27,44 for 10 months, at 3 per cent? *Ans.* \$,686.

9. What is the interest of \$1,98 for 7 years, at 5 per cent? *Ans.* \$,693.

10. What is the interest of \$756,17 for 2 months, at 12 per cent? \$15,1234.

ANOTHER METHOD OF COMPUTING INTEREST AT ALL RATES PER CENT.

RULE.—*Multiply one half the number of months and one sixth of the days by the rate per cent, and divide this product by 6. Multiply this quotient by the given sum of money, and the product will be the answer.*

1. What is the interest of \$49,78 for 3 years, 7 months, and 9 days, at 8 per cent?

3 yrs. 7 m.=43 m.$\div$2=215+,0015=,2165$\times$8=1,7320
1,7320$\div$6=,2886$\frac{4}{6}$$\times$49,78=\$14,369826$\frac{2}{3}$, *Ans.*

One half the number of months is 215, which are considered \$,215. One sixth part of the days is 15, which are considered \$,0015, which, added to the former sum, make \$,2165, the interest of one dollar for 3 years, 7 m. 9 d. at 6 per cent. The interest of one dollar (,2165) multiplied by 8, and divided by 6, gives the interest on one dollar for 3 years, 7 m. 9 d. at 8 per cent. The interest of one dollar at 8 per cent, multiplied by \$49,78, gives the interest, at 8 per cent, of \$49,78 for 3 years, 7 m. 9 d.

2. What is the interest of \$83,33$\frac{1}{3}$ for 1 year, 8 m. and 21 d. at 7 per cent? *Ans.* \$10,0652.

3. What is the interest of \$82,9 for 7 years, 11 months, 19 days, at 6$\frac{1}{4}$ per cent? *Ans.* \$41,291684027.

4. What is the interest of \$1000 for 3 years, 7 months, and 5 days, at 7$\frac{4}{5}$ per cent? *Ans.* \$280,583$\frac{1}{3}$.

5. What is the interest of \$62,50 for 1 year, 7 months, and 3 days, at 7 per cent? *Ans.* \$6,96354$\frac{1}{6}$.

TO COMPUTE INTEREST ON NOTES ON WHICH ENDORSEMENTS ARE MADE.

There are different methods of computing interest, according to the agreement of parties, and according to the laws of the different states.

The following rule is established by the Supreme Court of the state of Connecticut.

RULE.—*Compute the interest to the time of the first payment; if that be one year or more from the time the interest commenced, add it to the principal, and deduct the payment from the sum total. If there be after payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and, in like manner,*

from one payment to another; till all the payments are absorbed; provided the time between one payment and another be one year or more.

But if any payments be made before one year's interest hath accrued, then compute the interest on the principal sum due on the obligation for one year, add it to the principal, and compute the interest on the sum paid, from the time it was paid, up to the end of the year, add it to the sum paid, and deduct that sum from the principal and interest added together.

If any payments be made of a less sum than the interest arisen at the time of such payment, no interest is to be computed but only on the principal sum for any period.

Peterboro', June 1, 1828.

For value received, I promise to pay E. H. or order, nine hundred dollars, on demand, with interest. JAMES D. CASH.

$900.

On this note the following endorsements are made: June 16, 1829, received two hundred dollars of the within note. August 1, 1830, received one hundred and sixty dollars. November 16, 1830, received seventy-five dollars. Feb. 1, 1832, received two hundred and twenty dollars.

What was due, Aug. 1, 1832?

Ans. $444,9850588515625.

OPERATION,

Performed by the Standard rule at 6 per cent, in months.

Principal	$900,
Int. from June 1, 1828, to June 16, 1829, 12½ m.	56,25
	956,25
First payment	200,
Remainder for new principal	756,25
Int. fr. June 16, 1829, to Aug. 1, 1830, 13½ m.	51,046875
Operation carried forward ..	807,296875

Brought forward..	807,296875
Second payment.................	160,
Remainder for new principal.......	647,296875
Interest for 1 year................	38,8378125
New principal.....................	686,1346875
Amt. of 3d paym't fr. Nov. 16, 1830, to Aug. 1, 1831, 8½ months,.......	78,1875
Remainder for a new principal......	607,9471875
Int. fr. Nov. 16, 1830, to Feb. 1, 1832..	44,07617109375
Remainder for new principal	652,02335859375
Fourth payment...................	220,
Remainder for new principal.......	432,02335859375
Int. from Feb. 1, 1832, 6 months.....	12,9617002578125
Ans.	$444,9850588515625

XVIII. Compound Interest.

Compound Interest is computed by adding the interest to the principal annually, and making the principal and the interest the principal for the next year.

Rule.—*Divide the given sum by* 20, *and the quotient will be the interest at* 5 *per cent for one year. Divide the* 5 *per cent interest by* 5, *and the quotient will be* 1 *per cent, which, added to* 5 *per cent, makes* 6 *per cent. The* 6 *per cent interest, added to the principal, makes the amount; and this amount will be the principal for another year. Proceed in this manner till the close of the number of years. When parts of a year are given, cast the interest upon the given sum, for the given number of years, and upon this amount cast the interest for the parts of a year, and to the amount add this interest.*

1. What is the compound interest of $90 for 4 years, at 6 per cent?

20|90	
5|4,5	interest for 1 year, at 5 per cent.
9	interest for 1 year, at 1 per cent.
20|95,4	am't of $90 for 1 year, at 6 per cent.
5|4,77	interest of $95,4, 1 year, at 5 per cent.
,954	int. of $95,4 for 1 year, at 1 per cent.
20|101,124	am't of $95,4 for 1 year, at 6 per cent.
5|5,0562	int. of $101,124 for 1 yr. at 5 per cent.
1,01124	int. of $101,124 for 1 yr. at 1 per cent.
20|107,19144	amt. of $90 for 3 years, at 6 per cent.
5,359572	int. of $107,19144 1 yr. at 5 per cent.
1,0719144	int. of $107,19144 1 yr. at 1 per cent.
$113,6229264	amt. of $90 for 4 years, at 6 per cent.
$90,	principal subtracted from the amount.
Ans. $23,6229264	int. of $90, for 4 years, at 6 per cent, [compound interest.

2. What is the compound interest of $378,45 for 6 years, at 5 per cent? *Ans.* $138,83682335078125.

The pupil will readily perceive that interest, at all rates per cent, may be computed by the above rule, by adding to 5 per cent such per cent as will make the required per cent.

3. What is the compound interest of $36,36 for 4 years, at $8\frac{1}{3}$ per cent?

This rate per cent is $\frac{1}{12}$ of a hundred per cent, and the operation may be performed by dividing by 12.

12|36,36.
 3,03 interest of $36,36 for 1 yr. at 8⅓ per cent.

12|39,39 amount of $36,36 for 1 year.
 3,2825 interest of $39,39 for 1 year.

12|42,6725 amount of $36,36 for 2 years.
 3,556041⅔ interest of $42,6725 for 1 year.

12|46,228541⅔ amount of $36,36 for 3 years.
 3,852378$\frac{17}{36}$ interest of $46,228541⅔.

50,080920$\frac{5}{36}$
36,36 principal subtracted from the amount.

Ans. 13,720920$\frac{5}{36}$

4. What is the compound interest of $437,894 for 3 yrs. at 6 per cent? *Ans.* $83,644779484.

5. What is the compound interest of $400 for 5 years, at 10 per cent? *Ans.* $244,204.

6. What is the compound interest of $360,75 for 2 years, at 7½ per cent? *Ans.* $56,14671875.

7. What is the compound interest of $437,65 for 2 years, at 12½ per cent? *Ans.* $116,25078125.

8. What is the compound interest of $25, for 3 years, at 2½ per cent? *Ans.* $1,922265625.

Note.—In finding the time between two dates, reduce both dates to years and decimal parts of a year, and subtract the earlier date from the latter.

Interest should not be compounded oftener than once a year; but some compute the interest on a note to the time of an endorsement, and from the amount subtract the endorsement, which compounds the interest from that time. This practice is not correct.

XIX. Discount and Present Worth.

Discount is an allowance made for the payment of any sum of money before it becomes due, and is the difference between that sum due some time hence, and its present worth.

The present worth of any sum due some time hence, is such a sum as, when put at interest, would, in that time, and at the given rate per cent, amount to the sum then due.

TO FIND THE PRESENT WORTH OF ANY GIVEN SUM, FOR ANY GIVEN TIME.

Rule.—*Divide the given sum by the amount of one dollar for the given rate and time, and the quotient will be the present worth.*

1. What is the present worth of $63,6 due one year hence, discounting at the rate of 6 per cent?

Amount of 1 dollar, 1,06|63,6|60,

636 *Ans.* $60.

2. What is the present worth of $235,2 for 2 years, at 9 per cent? *Ans.* $$199,322\frac{2}{59}$.

TO FIND THE DISCOUNT ON ANY GIVEN SUM.

Rule.—*Subtract the present worth from the given sum, and the remainder will be the discount.*

1. What is the discount on $235,2, due 2 years hence, discounting at the rate of 9 per cent? *Ans.* $$35,877\frac{57}{59}$.

2. What is the discount on $100, due 1 year hence, discounting at the rate of 6 per cent? *Ans.* $$5,660\frac{20}{53}$.

3. What is the present worth of $28,126, due 10 months hence, discounting at 3 per cent? *Ans.* $27,44.

4. How much grain must be sent to mill, that a bushel of meal may be returned, the miller taking $\frac{1}{16}$ part for toll, and allowing that a bushel of grain would make only a bushel of meal? Ans. 1 bu. $2\frac{2}{15}$ qts.

OPERATION.

From 1 unit take $\frac{1}{16}$ of a unit, or ,0625.

$1-,0625=,9375.$

$1\div,9375=1$ bushel, $2\frac{2}{15}$ qts., *Ans*

Or thus: $1-\frac{1}{16}=\frac{15}{16}$.

$1\div\frac{15}{16}=1\frac{1}{15}=1$ bushel, $2\frac{2}{15}$ qts.

5. How much grain must be sent to mill, that 7 bushels and 7 quarts of meal may be returned, allowing the miller to take $\frac{1}{16}$ part for toll? And how much will the miller take for toll? *Ans.* 7 bushels, $22\frac{6}{15}$ qts. Toll, $15\frac{6}{15}$ qts.

TO FIND DISCOUNT ON ARTICLES SOLD FOR CASH, WHERE DISCOUNT IS MADE.

RULE.—*Subtract the rate per cent from $1, and by the remainder multiply the given sum, and the product will be the answer.*

1. A gentleman has in a bank $8796,49, and wishes to withdraw 20 per cent. How much will remain?

$1,—,2=,8

$8796,49×,8=$7037,192, *Ans.*

2. A gentleman had an estate of $15000; he gave $12\frac{1}{2}$ per cent to his wife; 25 per cent of the remainder to his children, and $16\frac{2}{3}$ per cent of the remainder to a charitable institution. What was the share of each, and how much had he left?

Ans.
- Wife's share $1875,
- Children's share .... $3281,25.
- Share for charitable institution,.. $1640,625.
- Sum left............................ $8203,125.

3. What is the difference between $12\frac{1}{2}$ per cent on $1000, and $33\frac{1}{3}$ per cent on $400? *Ans.* $8,33$\frac{1}{3}$.

4. A merchant expended $1800 as follows: he laid out 12 per cent of his money for calicoes; 20 per cent for hard ware; 40 per cent for broadcloths; 10 per cent for tea and coffee; and with the remainder he bought 3402 pounds of sugar. What was the sugar per pound? *Ans.* $,1.

THE PRICE AT WHICH ANY THING IS BOUGHT OR SOLD BEING GIVEN, TO FIND THE RATE PER CENT OF LOSS OR GAIN.

RULE.—*Divide the loss or gain by the price of the article, and the quotient will be the rate per cent.*

1. A merchant buys goods to the amount of $1250, and sells them so as to gain $275. What rate per cent did he gain?

275÷1250=2200÷10000=,22.

Ans. 22 per cent.

This division is performed by multiplication.

2. I bought a horse for $166,66⅔, and sold him for $187,5. What was the gain per cent?

187,5
166,66⅔

20,83⅓ | 166,66⅔, divisor.

Ans. $,125, or 12½ per cent.

3. A tax on a certain town is $1627,18, on which the collector is to receive 2½ per cent for his services. How much must he receive? *Ans.* $40,6795.

4. A collector received $40,6795 for collecting $1627,18. What per cent did he receive? *Ans.* 2½ per cent.

5. A merchant bought goods to the amount of $3786,875. What must he sell them for, to gain 15 per cent?

Ans. $4354,90625.

As the method of operation in Commission, Brokerage, Stocks, Insurance and Policies, and Banking is the same as in Interest and Discount, no separate notice of these topics is necessary.

THE INTEREST, TIME, AND RATE PER CENT GIVEN, TO FIND THE PRINCIPAL.

RULE.—*Divide the number of days by the interest, and by this quotient divide the standard for the given rate, and the quotient will be the answer.*

1. What sum, at 6 per cent, will gain \$4 in 96 days?

4|96
———
24|6000. .the standard at 6 per cent.
———
Ans. 250

2. What sum will gain \$40 in 1200 days, at 6 per cent? *Ans.* \$200.

3. What sum will gain \$5,5 in 495 days, at 7 per cent? *Ans.* \$57,142857$\frac{1}{7}$.

THE PRINCIPAL, INTEREST AND TIME GIVEN, TO FIND THE RATE PER CENT.

RULE.—*Divide the number of days by the interest, and by the quotient multiply the given sum, and the product will be a standard in days in which the money would double. By this standard divide the 1 per cent standard, and the quotient will be the rate per cent.*

1. If \$250 gain \$4 in 96 days, what is the rate per cent?

```
4|96        250
———          24
 24        ————
           1000
           500
           ————
           6000|36000, 1 per cent standard.
                Ans. 6.
```

2. If \$36 gain \$1,08 in 8 months, what is the rate per cent? *Ans.* 4$\frac{1}{2}$.

3. If \$120 gain \$,98 in 49 days, what is the rate per cent? *Ans.* 6 per cent.

4. If \$14,4 gain \$,0948 in 79 days, what is the rate per cent? *Ans.* 3 per cent.

5. If \$75 gain \$6,75 in one year and six months, what is the rate per cent? *Ans.* 6 per cent.

6. If \$120000 gain \$150 in 7$\frac{1}{2}$ days, what is the rate per cent? *Ans.* 6 per cent.

7. If $75 gain $150 in 120000 days, what is the rate per cent? *Ans.* 6 per cent.

THE PRINCIPAL, INTEREST AND RATE PER CENT BEING GIVEN, TO FIND THE TIME.

Rule.—*Divide the given interest by the interest on the principal for one year, and the quotient will be the time, in years and decimal parts of a year.* Or,

Divide the principal by the interest, and by this quotient divide the standard, and the quotient will be the time in days.

1. How long must $500 be on interest to gain $120?

$120 \div 30 = 4$ years, *Ans.*

2. How long must $120 be on interest, at $8\frac{1}{4}$ per cent, to gain $133,2? *Ans.* 13 yrs. 5 m. 12 d. 15 h. 1 m. $49\frac{9}{99}$ s.

3. How long must $280 be on interest at $7\frac{1}{4}$ per cent, to gain $411,95? *Ans.* $20{,}2931\frac{7}{203}$ yrs.

When the sum of money comes on the standard, the time in days may be found by multiplying the interest by the same that we divide by, in finding the interest, and the product will be the time in days.

TIME, RATE PER CENT, AND AMOUNT GIVEN, TO FIND THE PRINCIPAL.

Rule.—*Divide the given amount by the amount of $1 for the given rate and time, and the quotient will be the principal.*

1. What principal, at 8 per cent, in 1 year and 6 months, will amount to $85,12? *Ans.* $76.

2. What principal, at 6 per cent, in 11 months and 9 days, will amount to $99,311? *Ans.* $94.

3. What principal, at 9 per cent, in 3 yrs. 7 m. 21 days, will amount to $442,58$\frac{1}{3}$? *Ans.* $333,33$\frac{1}{3}$.

XX. Equation of Payments.

Equation of Payments is finding the time in which several payments, due at different times, should be paid at once, without gain or loss to the debtor or creditor.

RULE.—*Compute the interest on all the several sums, for their respective times; add the several interests; and the time in which the amount of the several sums will gain the amount of the several interests, will be the true equated time.*

1. A owes B $19, $5 of which are to be paid in 6 m., $6 in 7 m., and $8 in 10 months; what is the equated time for the payment of the whole?

The interest of	$5,	6 m.	is $,15
The interest of	$6,	7 m.	is $,21
The interest of	$8,	10 m.	is $,4
	$19,		$,76

The time in which $19 will gain $,76 will be the equated time. *Ans.* 8 months.

2. A owes B $300, $50 of which are to be paid in 2 months; $100 in 5 months; the remainder in 8 months. What is the equated time for the payment of the whole sum? *Ans.* 6 months.

3. A owes B $150, $50 of which are to be paid in 4 months, and $100 in 8 months. B owes A $250, to be paid in 10 months. It is agreed between them, that A shall make present payment of his whole debt, and that B shall pay his so much sooner as to balance the favor. I demand the time at which B must pay the $250.

The interest of $50, 4 months, is $1,
The interest of $100, 8 months, is $4,

Whole amount of int. on this is $5,
The interest of $250, 10 months, is $12,5
Subtract the first interest from the last, 5,

$7,5

In what time will $250 gain $7,5?

Ans. 6 months, equated time.

4. A merchant has $144 due to him, to be paid in 7 months; but the debtor agrees to pay $\frac{1}{2}$ ready money, and $\frac{1}{3}$ in 4 months. How long shall the remainder be kept after the second payment is made? *Ans.* 2 yrs. 5 m.

5. $3000 are due to a merchant; $200 of which are to be paid in 3 months; $300 in 5 months, and the remainder in 10 months. What is the equated time for the payment of the whole sum? *Ans.* 7 months and 3 days.

XXI. Square and Cubic Measure.

Under this head, the method of performing all problems in duodecimals will be given.

TO MEASURE BOARDS, OR SURFACES OF UNIFORM DIMENSIONS.

RULE.—*Multiply the length by the breadth.*

1. How many feet in a board 4 feet in length, and 6 inches in width?

4	Or thus	4 ft.=48 in.
,5		6 in.
2,0 *Ans.* 2 ft.		144|288
		Ans. 2 feet.

The last process may be explained by the following formula: $\frac{48}{12}\times\frac{6}{12}=\frac{288}{144}=2$ feet.

It will be readily seen, that the operation may be performed by the rule already given for multiplying parts and parts of parts by parts and parts of parts.

2. How many feet in a board 17 ft. 7$\frac{5}{8}$ in. long, and 2 ft. 5$\frac{3}{8}$ in. wide?

17 ft. 7$\frac{5}{8}$ in.=211,625 in. or 12th parts.
2 ft. 5$\frac{3}{8}$ in.= 29,375 in. or 12th parts.

$\frac{211,625}{12}\times\frac{29,375}{12}=\frac{6216,484375}{144}=$43 ft. 2 pr. 5$\frac{13}{16}$ in., *Ans.*

3. How many feet in a board 11 feet, 9 inches long, and 1 foot, 3 inches wide?

11 ft. 9 in.=11,75		Or thus:	8|11,75
1 ft. 3 in.= 1,25			1,25
5875		*Ans.*	14,6875
2350			
1175			
Ans. 14,6875			

4. How many square feet in a board 16 feet and 7 in. long, and 1 foot and $1\frac{1}{2}$ in. wide?

16 ft. 7 in.= 199 in.
1 ft. $1\frac{1}{2}$ in.= 13,5 in.

2487,5. .prod. by 12,5.
199, . .prod. by 1.

12|2686,5. .10,5 in. or seconds, rem.

12|223,. .7 primes remainder.

18 feet.

Ans. 18 ft. 7 primes, and 10,5 in.

5. How many square inches in a strip of board 2 ft. 2 in. and 6 seconds long, and 3 in. 6 seconds wide?

Ans. 92,75 in.

6. How many square inches in a board 7 ft. 7 in. long, and 1 ft. $4\frac{2}{3}$ in. wide? *Ans.* $1516\frac{2}{3}$ in.

TO MEASURE BOARDS, OR SURFACES WIDER AT ONE END THAN AT THE OTHER.

RULE.—*Add the measures of the ends, and divide the amount by 2, and the quotient will be the average width. Proceed, then, according to the rule given to measure surfaces of uniform dimensions.*

1. What are the dimensions of a board 13 ft. 8 in. long, and 17 in. wide at one end, and 18 in. at the other?

17+13÷2=15..average width.
13 ft. 8 in.=41..3d parts.
15 in.= 5..4th parts.

12|205

Ans. 17 ft. and 1 prime.

2. What are the dimensions of a board 17 ft. 2 in. long, and 19,5 in. wide at one end, and 13,75 in. at the other? *Ans.* 23 ft. 9$\frac{1}{16}$ primes.

The average width of a board may be found also by measuring the width of the board in the centre. It is understood, of course, that the diminution is gradual from the wider end to the narrower.

3. Required the number of feet in a stock of 15 boards, 12 ft. 8 in. long, and 13 in. wide. *Ans.* 205 ft. 10 primes.

4. What are the dimensions of 17 boards, 19 ft. 11 in. long, 2 ft. 5 in. wide at one end, and 1 ft. 9 in. at the other? *Ans.* 705 ft. 4 primes, 7 in.

THE WIDTH OF ANY THING GIVEN, TO FIND WHAT MUST BE THE LENGTH, TO MAKE ANY GIVEN QUANTITY.

RULE.—*Divide the given quantity by the width.*

1. If a board be 4 in. wide, what must be its length, to make one square foot? 144÷4=*Ans.* 36 in.

2. If a board be 1$\frac{1}{4}$ in. wide, how long must it be to make one square foot? *Ans.* 115,2 in.

3. If a floor be 83$\frac{1}{3}$ in. wide, what must be its length to make 100 feet? *Ans.* 172,8 in.

It will be readily seen, that when the *length* and *quantity* of any thing are given to find the width, the quantity divided by the length will give the width.

THE LENGTH, BREADTH, AND THICKNESS GIVEN, TO FIND THE CUBICAL OR SOLID CONTENTS.

RULE.—*Multiply the length by the breadth, and that product by the thickness.*

1. How many solid feet in a block 15 ft. 8 in. long, 1 ft. 5 in. wide, and 1 ft. 4 in. thick?

15 ft. 8 in. = 188 in. or 12th parts.
1 ft. 5 in. = 17 in. or 12th parts.

```
                 1316
                 188
                 ----
                 3196..144th parts.
1 ft. 4 in. =      16 in. or 12th parts.
                 -----
                 19176
                 3196
                 -----
              12|51136..1728th parts.   4 in. or sec.
               12|4261..1 prime.
                 12|355..7 ft. board measure.
                    29 ft.
```

Ans. 29 ft. 7 ft. b. m. 1 pr. 4 in.

2. What are the solid contents of a block of marble, 10 ft. long, 5 ft. 9 in. wide, and 3 ft. 6 in. thick?

$10 \times 5,75 \times 3,5 =$ *Ans.* 201,25 feet.

3. How many cubical inches in a bin of grain, 17 ft. 7⅝ in. long, 11 ft. 11⅞ in. wide, and 6 ft. 7¾ in. high?

Ans. 2428191,86328125.

4. How many solid feet in a pile of wood, 18 ft. 6 in. long, 2 ft. 4 in. wide, and 2 ft. 3 in. thick?

Ans. 97 ft. 1 ft. b. m. 6 pr.

HAVING TWO PARTS OF A CUBICAL FORM AND THE CUBICAL CONTENTS GIVEN, TO FIND THE THIRD PART.

RULE.—*Divide the cubical contents by the product of the two given parts, and the quotient will be the third part.*

1. How long must a stick of scantling be to make a solid foot, which is 4 in. wide, and 3 in. thick?

$1728 \div 12 =$ *Ans.* 144 in. = 12 ft.

2. How high must a pile of wood be to make 3 cords and 6 cord feet, that is 40 feet long, and 3 feet wide?

Ans. 4 feet.

3. How long must a pile of wood be to contain 10 cords, that is 5 feet high, and 5 feet wide? *Ans.* 51,2 feet.

4. How long must a pile of wood be to contain 9 cords, that is 6 ft. 4 in. wide, and 4 ft. 6 in. high?

Ans. 41$\frac{49}{55}$, or 41,89$\frac{1}{11}$ ft.

XXII. Proportion.

Proportion is composed of *ratios.* A ratio is composed of *antecedent* and *consequent.* The *first* term of a ratio is called the *antecedent,* and the *second* term the consequent.

The ratio which one number bears to another, may be found by dividing the second term, or consequent, by the first term, or antecedent.

Thus, the relation which 8 has to 4, (8 : 4) may be found by dividing 4 by 8, $\frac{4}{8}=\frac{1}{2}$.

In a proportion, there are two ratios, two *antecedents* and two *consequents.* Thus, 8 : 4 :: 12 : 6. The antecedents in this proportional sum are 8 and 12, and the consequents are 4 and 6.

In single proportion, three terms are given to find the fourth; that is, one ratio is given, and the antecedent of another. And because the extremes of any proportional sum multiplied together, produce a sum equal to the product of the means, it is plain that the product of the means, divided by the given extreme, will give the other extreme. Thus, 8 : 4 :: 12 : 6.

$4\times 12\div 8=6$, the consequent of the second ratio.

The first and last terms, in a proportion, are called *extremes,* and the other two *means.*

Proportion is divided into *single* and *double.*

TO STATE A SUM IN SINGLE PROPORTION.

RULE.—*Make that number which is of the same kind of*

the answer the third term, or antecedent of the second ratio. Take the two remaining numbers that enter into the sum, and make the larger number the second term, or consequent of the first ratio, and the other number, the first term, or antecedent of the first ratio, if the answer is to be larger than the third term. If the answer is not to be larger than the third term, make the larger of the two remaining numbers the antecedent of the first ratio, and the smaller one the consequent of the same ratio. Multiply the second and third terms together, and divide by the first, and the quotient will be the answer.

Note.—It must be remembered that the first and second terms of a proportion must be of the same denomination. Therefore, if they are not of the same kind, in the given sum, they must be reduced to the same denomination, before they are introduced into the proportion.

1. If ,75 of a unit give 6,75 units, how much will 1,25 units give?

,75 : 1,25 : : 6,75

OPERATION.

```
8|6,75
   1,25
 ---------
   8,4375
 ---------
        4
 ---------
3|33,7500
 ---------
Ans. 11,25
```

The pupil will perceive that I multiplied by 1,25 by dividing by 8, and that I divided by ,75 by using $\frac{3}{4}$.

2. If $1\frac{1}{4}$ yd. in breadth required $20\frac{1}{2}$ yds. in length, to make a cloak, what in length that is $\frac{3}{4}$ yd. wide will be required to make the same?

1. By eighths. ,75 : 1,25 : : 20,5

```
6|205
-----
Ans. 34⅙
```

2. By 4ths.

,75 : 1,25 : : 20,5
5

3|102,5

Ans. 34,16$\frac{2}{3}$=34$\frac{1}{6}$.

3. By 12ths.

,75 : 1,25 : : 20,5
15

1025
205

9|307,5

Ans. 34,16$\frac{2}{3}$

4. By 16ths.

,75 : 1,25 : : 20,5
20

12|410,0

Ans. 34$\frac{1}{6}$

5. By 6ths.

,75 : 1,25 : : 20,5
7,5

1025
1435

4,5|153,75|34,16$\frac{2}{3}$
135

187
180

75
45

300
270

$\frac{30}{45}$|15|$\frac{2}{3}$

Many more variations may be had on this sum by multiplying and dividing by those numbers that do not alter the ratios in the given sum. The same principle applies in all sums in proportion, or any other rule.

3. If $5 buy 9 bushels and 12 quarts of oats, how many bushels will $15 buy? *Ans.* 28 bu. 4 qts.

4. If 6 horses consume 21 bushels of oats in 4,5 weeks, how many bushels will 20 horses consume in the same time? *Ans.* 70 bu.

5. If 9 men build 17$\frac{1}{16}$ rods of wall in 10 days, how many rods of wall would 108 men build in the same time? *Ans.* 204$\frac{3}{4}$ rods.

6. If 2 feet and 1 inch of staff cast a shadow 2 feet and 9$\frac{1}{3}$ inches, how high is the steeple that casts a shadow 97 feet and 11 inches? *Ans.* 130[illegible] feet.

7. If 6,25 acres of land cost \$3125, what will 812,5 acres cost? *Ans.* \$406250.

8. If 3 and 4 make 9, how many will 6 and 8 make? *Ans.* 18.

9. If $\frac{1}{3}$ of $\frac{18}{20}$ be $47\frac{1}{2}$, what will $\frac{1}{4}$ of 150 be? *Ans.* $59\frac{3}{8}$=59,375.

10. If $\frac{1}{3}$ of 6 be 3, what will $\frac{1}{4}$ of 20 be? *Ans.* 7,5.

11. If $\frac{3}{4}$ of a pound cost $\frac{5}{8}$ of a dollar, what will $\frac{9}{10}$ of a pound cost? *Ans.* \$,75.

12. The earth, being 360° in circumference, turns round on its axis in 24 hours; how far does it turn in one minute, in the 43d parallel of latitude; the degree of longitude, in this latitude, being about 51 statute miles? *Ans.* $12\frac{3}{4}$ mi.

13. A merchant bought a number of bales of velvet, each containing $129\frac{17}{27}$ yds. at the rate of \$7 for 5 yds., and sold them out at the rate of \$11 for 7 yds., and gained \$200 by the bargain; how many bales were there? *Ans.* 9.

COMPOUND PROPORTION

is composed of two ratios, and the antecedent of a third or more.

There may be many more ratios than I have here mentioned, in some cases. Thus:

$$\left.\begin{array}{r@{\;:\;}l} 24 & 6 \\ 8 & 9 \\ 20 & 200 \\ 6 & 8 \\ 4 & 6 \end{array}\right\} :: 16 : 90$$

The antecedents of the first ratios are multiplied together for a divisor, and the consequents of the same ratios are multiplied together, and that product by the antecedent of the second ratio, for a dividend, and the quotient will be the answer.

TO STATE A SUM IN COMPOUND PROPORTION.

RULE.—*Make that number which is like the answer the*

third term. Arrange the other numbers that enter into the proportion, according to the rule in simple proportion. Multiply all the numbers in the second place together, and that product by the third term, for a dividend. Multiply all the numbers in the first place for a divisor, and the quotient arising from this division will be the answer.

1. If £100 in 12 months gain £6, what will £25 gain in 4 months?

$$\left.\begin{array}{r} 100 : 25 \\ 12 : \;\;4 \end{array}\right\} : : 6$$

$$\begin{array}{r} \overline{1200} : \overline{100} : : 6 \end{array}$$

$$100\times6\div1200=£\,,5=10\text{ s., } \textit{Ans.}$$

2. If 120 bushels of corn can serve 14 horses 56 days, how many days will 94 bushels serve 6 horses?

Ans. $102\frac{16}{45}$ days.

3. If 7 oz. 5 pwt. of bread be bought for $4\frac{3}{4}$ d. when corn is 4 s. 2 d. per bushel, what weight of it may be bought for 1 s. 2 d. when the price per bushel is 5 s. 6 d.?

Ans. 1 lb. 4 oz. $3\frac{478}{627}$ pwt.

4. What principal will gain £262 10 s. in 7 years, at £5 per cent per annum? *Ans.* £750.

5. If 12 men, in 15 days, can build a wall 30 feet long, 6 feet high, and 3 feet thick, when the days are 12 hours long, in what time will 60 men build a wall 300 feet long, 8 feet high, and 6 feet thick, when they work only 8 hours a day? *Ans.* 120 days.

6. How long will it take $500 to gain $10, if $100 gain $6 in one year? *Ans.* 4 months.

7. If 3 men receive £$8\frac{9}{10}$ for $19\frac{1}{2}$ days' work, how much must 20 men receive for $100\frac{1}{4}$ days' work?

Ans. £305 0 s. 8 d.

8. If 4 reapers receive $11,04 for 3 days' work, how many men may be hired 16 days for $103,04?

Ans. 7 men.

9. If 8 men spend £32 in 13 weeks, what will 24 men spend in 52 weeks? Ans. £384.

10. There was a certain edifice completed in a year by 20 workmen; but the same being demolished, it is necessary that just such a one should be built in 5 months. I demand the number of men to be employed about it.

Ans. 48 men.

11. If the freight of 9 hhds. of sugar, each weighing 12 cwt., 20 leagues, cost £16, what must be paid for the freight of 50 tierces, each weighing $2\frac{1}{2}$ cwt., 100 leagues?

Ans. £92 11 s. $10\frac{2}{3}$ d.

12. If 950 soldiers consume 350 quarters of wheat in 7 months, how many soldiers will consume 1464 quarters in 1 month? *Ans.* 27816 soldiers.

13. If 1464 quarters of wheat be used by 27816 soldiers in a month, in what time will 950 soldiers consume 350 quarters? *Ans.* 7 months.

14. If $100 gain $6 in a year, what will $400 gain in 9 months? *Ans.* $18.

15. If $100 will gain $6 in a year, in what time will $400 gain $18? *Ans.* 9 months.

XXIII. Value of Compounds or Mixtures.

TO FIND THE MEDIUM PRICE OF SEVERAL ARTICLES MIXED, THE QUANTITY AND VALUE OF EACH BEING GIVEN.

RULE.—*Divide the whole cost of the articles by the sum of the articles, and the quotient will be the medium price of one article.*

1. A grocer mixed 2 cwt. of sugar, at $9 per cwt. and 1 cwt. at $7 per cwt. and 2 cwt. at $10 per cwt.; what is the value of 1 cwt. of this mixture?

2	cwt. at	$9=	$18,
1	"	7=	7,
2	"	10=	20,
5			45÷5=*Ans.* $9.

2. If 3 pounds of gold, of 22 carats fine, be mixed with 3 pounds, of 20 carats fine; what is the fineness of the mixture?

$$\begin{array}{r} 22\times3=66 \\ 20\times3=60 \\ \hline 6\,|\,126 \\ \hline \textit{Ans.}\ 21 \end{array}$$

3. If I mix 10 lbs. of sugar, worth \$,1 per lb.; 8 lbs. worth \$,12; 20 lbs. worth \$,14; what must I charge per pound of the mixture? *Ans.* \$ $,125\frac{5}{19}$.

TO FIND WHAT QUANTITY OF EACH OF THE INGREDIENTS WHOSE RATES ARE GIVEN, WILL COMPOSE A MIXTURE OF A GIVEN RATE.

RULE.—*Place the several prices of the ingredients in a column under each other, the least uppermost, and so on, downward, according to their value.*

Connect, with a continued line, the price of each ingredient which is less than the rate of the compound, with one which has a greater value than the compound. Place the difference between the mean price, or rate of the compound, and that of each ingredient, opposite to the rates with which they are connected. If only one difference stand against any rate, it will be the quantity belonging to that rate; but if there be several, their sum will be the quantity.

1. A merchant has spices, some at 1 s. 6 d. per lb., some at 2 s., some at 4 s., and some at 5 s. per lb.; how much of each sort must he mix, that he may sell the mixture at 3 s. 4 d. per lb.?

	d.	lb.	
Medium rate, 40 d.	18	20 at 1 s. 6 d.	per lb.
	24	8 at 2 s.	"
	48	16 at 4 s.	"
	60	22 at 5 s.	"

By connecting the less rate with the greater, and placing the difference between them and the medium rate alternately, or one after the other in turn, the quantities resulting are such that there is precisely as much gained by one quantity as is lost by the other, and therefore the gain and loss, upon the whole, are equal and exactly the proposed rate. Various answers arise from the different modes of uniting the rates of the ingredients.

2. How much wine, at 6 s. per gallon, and at 4 s. per gallon, must be mixed together, that the compound may be worth 5 s. per gallon? *Ans.* 1 gal. of each.

3. How much corn, at 2 s. 6 d., 3 s. 8 d., 4 s., and 4 s. 8 d. per bushel, must be mixed together, that the compound may be worth 3 s. 10 d. per bushel?

Ans. 12 at 2 s. 6 d., 12 at 3 s. 8 d., 18 at 4 s., and 18 at 4 s. 8 d.

TO FIND THE SEVERAL QUANTITIES WHEN ONE OF THE INGREDIENTS IS LIMITED TO A CERTAIN QUANTITY.

RULE.—*Find the differences between the medium rate and the price of each ingredient, according to the preceding rule. Then say, as the difference of that simple, whose quantity is given, is to the rest of the differences severally, so is the quantity given, to the several required quantities.*

1. How much wine at 5 s., at 5 s. 6 d., and at 6 s. the gallon, must be mixed with 3 gallons, at 4 s. per gallon, so that the mixture may be worth 5 s. 4 d. per gallon?

$$
64\left\{\begin{array}{l}\text{s.}\\48\\60\\66\\72\end{array}\right. \qquad \begin{array}{r} \\ -8+2=10\\ -8+2=10\\ -16+4=20\\ -16+4=20\end{array}
$$

$$
10 : \left\{\begin{array}{c}10\\20\\20\end{array}\right\} :: 3 : \left\{\begin{array}{c}3\\6\\6\end{array}\right.
$$

Ans. 3 gal. at 5 s., 6 at 5 s. 6 d., and 6 at 6 s.

2. A grocer would mix teas at 12 s., 10 s., and 6 s., with 20 lb. at 4 s. per lb.; how much of each sort must he take to make the compound worth 8 s. per lb.?

Ans. 20 lb. at 4 s., 10 lb. at 6 s., 10 lb. at 10 s., and 20 lb. at 12 s.

TO FIND THE SEVERAL QUANTITIES WHEN THE WHOLE COMPOUND IS LIMITED TO A CERTAIN QUANTITY.

RULE.—*Find the proportional quantities, and then say, as the sum of the proportional quantities, or differences, is to the given quantity, so is each proportional quantity, or difference, to the required quantity of each.*

1. How many gallons of water must be mixed with wine worth 3 s. per gallon, so as to fill a vessel of 100 gallons, and that a gallon may be afforded at 2 s. 6 d.?

$$30\begin{cases}0 & 6\\ 36 & \underline{30}\\ & 36\end{cases}$$

$$36 : 100 :: \begin{cases}6\\ 30\end{cases} : \begin{cases}16\frac{2}{3}\\ 83\frac{1}{3}\end{cases}$$ *Ans.* $83\frac{1}{3}$ wine, and $16\frac{2}{3}$ water.

2. A grocer has currants at 4 d., 6 d., 9 d., and 11 d. per lb., and he would make a mixture of 240 lb. so that it might be afforded at 8 d. per lb.; how much of each sort must he take?

Ans. 72 lb. at 4 d., 24 lb. at 6 d., 48 lb. at 9 d., and 96 lb. at 11 d.

XXIV. Partnership.

The object of this section is to estimate the gain or loss of individuals doing business in partnership. The processes here given, hardly require a separate consideration; but in conformity to long-established usage, rather than from a conviction of propriety, a method of working partnership-sums is given.

TO FIND THE SHARE OF GAIN OR LOSS OF EACH INDIVIDUAL, WHEN HIS SHARE IS EXPRESSED BY A FRACTION.

RULE.—*Reduce the fractions to the same denominator, and divide the whole gain or loss by the denominator, and multiply the quotient by each numerator, and the several products will be the several answers.*

1. A and B companied, and trade with a joint capital of $400; A receives, for his share of the gain, one half as much as B; what was the stock of each?

It is plain that A receives $\frac{1}{3}$, and B $\frac{2}{3}$. Therefore, divide $400 by the denominator 3, and multiply by the numerators.

$\$400 \div 3 =$ A's $\$133\frac{1}{3} \times 2 =$ B's $266\frac{2}{3}$.

2. Divide $125 between two men in such a manner that one shall have $\frac{1}{3}$ more than the other, after the sum is divided. *Ans.* One has $50, and the other $75.

TO FIND THE SHARE OF GAIN OR LOSS WHEN EACH INDIVIDUAL IS TO RECEIVE ACCORDING TO HIS STOCK IN TRADE.

RULE.—*Divide the whole gain or loss by the whole stock in trade, and multiply the quotient by each man's share of the stock.*

When the whole gain or loss is divided by the whole stock in trade, the quotient is the gain or loss on one dollar. Then the gain or loss of each man will be found by multiplying the gain or loss on one dollar by the number of dollars which constitute his share.

1. A, B, C, and D are concerned in a joint stock of $1000, of which A's part is $150, B's $250, C's $275, and D's $325. Upon the adjustment of their accounts, they have lost $337,5. What is the loss of each?

337,5 ÷ 1000 = ,3375, loss on one dollar.

,3375 × 150, A's share = $50,625, A's loss.
,3375 × 250, B's share = $84,375, B's loss.
,3375 × 275, C's share = $92,8125, C's loss.
,3375 × 325, D's share = $109,6875, D's loss.

2. A and B gained by trading $182. A put into stock $300, and B $400. What is the gain of each?

Ans. A's gain, $78, B's gain, $104.

3. A merchant failing in trade, owes A $600, B $760, C $840, D $800. His effects are sold for $2275. What will each receive of the dividend?

Ans. { A will receive $455,
B............576,33$\frac{1}{3}$
C............637,
D............606,66$\frac{2}{3}$ }

TO FIND THE GAIN OR LOSS OF INDIVIDUALS DOING BUSINESS IN PARTNERSHIP, WHEN THEIR STOCK CONTINUES IN TRADE DURING UNEQUAL TIMES.

RULE.—*Multiply each man's share of the stock by the time it continued in trade, and proceed according to the preceding rules.*

1. Three graziers hired a piece of land for $60,5. A put in 5 sheep for $4\frac{1}{2}$ months, B put in 8 for 5 months, and C put in 9 for $6\frac{1}{2}$ months; how much must each pay of the rent?

$5\times4\frac{1}{2}=22,5$
$8\times5\ =40,$
$9\times6\frac{1}{2}=58,5$

121,|60,5|,5
605

$,5\times22,5=\$11,25$, A's share.
$,5\times40,\ =\$20,$ B's share.
$,5\times58,5=\$29,25$, C's share.

2. Two persons hired a coach to go 40 miles, for $20, with permission to take in two more when they pleased. Now, when they had gone 15 miles, they admit C, who wished to go the same route; and on their return, within 25 miles of home, they admit D for the remainder of the journey. Now, as each person was to pay in proportion to the distance he rode, it is required of *you* to settle the coach-hire between them. *Ans.* A and B, $6,4 each, C, $5,2, and D, $2.

3. Two merchants enter into partnership for 18 months; A put into stock at first \$200, and at the end of 8 months he put in \$100 more; B put in at first \$550, and at the end of 4 months took out \$140. Now, at the end of the time they find they have gained \$526; what is each man's share?

Ans. A's, \$192,95$\frac{70}{1254}$, B's, \$333,04$\frac{1184}{1254}$.

4. A, with a capital of \$1000, began trade, Jan. 1, 1776, and meeting with success in business, he took in B a partner, with a capital of \$1500, on the 1st of March following. Three months after that, they admit C as a third partner, who brought into stock \$2800; and after trading together till the first of the next year, they find the gain, since A commenced business, to be \$1776,5. How must this be divided among the partners?

Ans. $\left\{\begin{array}{l}\text{A's \$457,40}\frac{364}{466}. \\ \text{B's \$571,83}\frac{222}{466}. \\ \text{C's \$747,19}\frac{346}{466}.\end{array}\right.$

XXV. Square Root.

The product of any number multiplied by itself, is called a *square*. When considered in relation to this product, the given number is called a *square root*.

Thus, $12 \times 12 = 144$, the square of 12.

But 12, considered as one of the factors of 144, are the *square root* of 144. Square numbers are found by multiplying any numbers by themselves. It will be found by experiment, that the square of any number contains twice as many figures as the root, or one less than twice as many. Thus, $9 \times 9 = 81$, the square of 9. But $10 \times 10 = 100$, the square of 10.

The square of a fraction is found by multiplying the numerator by itself, for the square of the numerator, and the denominator by itself, for the square of the denominator.

$$\text{Thus, the square of} \left\{\begin{array}{c}\frac{1}{3}\\ \frac{2}{4}\\ \frac{3}{5}\\ \frac{5}{9}\\ \frac{7}{10}\end{array}\right\} \text{is} \left\{\begin{array}{l}\frac{1}{9};\\ \frac{4}{16};\\ \frac{9}{25};\\ \frac{25}{81};\\ \frac{49}{100}, \text{ \&c.}\end{array}\right.$$

To find the square root of any number, is to find a number which, multiplied by itself, will produce the given number.

TO FIND THE SQUARE ROOT.

RULE.—*Beginning at the unit figure, distinguish the sum into periods of two figures each, by placing a dot over the unit figure, another over hundreds, and so on. If the sum contain decimals, place a dot over hundredths and tenths of thousandths, and so on.*

Find, by trial, the greatest square of the left-hand period, and place its root at the right hand of the given sum. Subtract the greatest square of the left-hand period from that period, and to the remainder annex the second period, and call them the first dividend. Divide this dividend (except the right-hand figure) by twice the root of the greatest square of the left-hand period, and place the quotient at the right hand of the root before found, and also at the right hand of the divisor. Multiply the divisor, with the figure annexed, by the last figure of the root, and subtract the product from the first dividend. To the remainder annex the third period, and call them the second dividend. Divide the second dividend (except the right-hand figure) by the first figure of the root and twice the last figure, (as one divisor,) and place the quotient figure at the right hand of the figures in the root already found, and also at the right hand of the divisor, and proceed as before, until the whole root is found.

1. What is the square root of 1936?

$$
\begin{array}{r|l}
 & \dot{1}9\dot{3}6 \,|\, 44, \textit{ Ans.} \\
 & 16 \\
\hline
84 & 336 \\
 & 336 \\
\hline
\end{array}
$$

2. What is the square root of 6561?

```
   6561|81, Ans.
   64
   --
161|161
    161
    ---
```

3. What is the square root of 103041?

```
  103041|321, Ans.
  9
  -
 62|130   first dividend.
    124
    ---
  641|641   second dividend.
      641
      ---
```

4. What is the square root of 28,7296?

```
    28,7296|5,36, Ans.
    25
    --
103|3,72   first dividend.
    309
    ---
 1066|6396   second dividend.
      6396
      ----
```

5. What is the square root of 13,3225? *Ans.* 3,65.

6. What is the square root of 806404? *Ans.* 898.

7 What is the square root of $\frac{25}{100}$? *Ans.* $\frac{5}{10}$.

8. What is the square root of $\frac{16}{64}$? *Ans.* $\frac{4}{8}=\frac{1}{2}$.

After the square root of a fraction is found, the *value* of that root may sometimes be expressed in lower terms than those of the root. But the fraction must *not* be reduced to its lowest terms *before* finding the square root. Although the *value* of the root would be the same in each case, yet the *expression* would be very different.

9. What is the square root of $11\frac{1}{9}$? *Ans.* $3\frac{1}{3}$.

In finding the root of a mixed number, reduce the mixed number to an improper fraction, and find the root of the numerator and denominator, and then reduce the fraction to a whole or mixed number.

10. What is the square root of $74\frac{71}{121}$? *Ans.* $8\frac{7}{11}$.
11. What is the square root of $38\frac{11}{25}$? *Ans.* $6\frac{1}{5}$.
12. What is the square root of 10,5625? *Ans.* 3,25.
13. What is the square root of 20,25? *Ans.* 4,5.
14. What is the square root of 72,25? *Ans.* 8,5.
15. What is the square root of $10\frac{15}{121}$? *Ans.* $3\frac{2}{11}$.
16. What is the square root of 390625? *Ans.* 625.
17. What is the square root of 765625? *Ans.* 875.
18. What is the square root of ,00390625? *Ans.* ,0625.
19. What is the square root of ,000009? *Ans.* ,003.
20. What is the square root of ,0016? *Ans.* ,04.
21. What is the square root of 36,048016? *Ans.* 6,004.
22. What is the square root of 9,1809? *Ans.* 3,03.
23. What is the square root of 16,6464? *Ans.* 4,08.
24. What is the square root of 105625? *Ans.* 325.
25. What is the square root of 10,5625? *Ans.* 3,25.

Having shown the scholar *how* sums are performed in the square root, I wish him now to attend carefully to the following explanation of the rule, that he may understand the nature of the process, and thus be able to give the *why* in relation to the several steps of the rule.

Suppose a man is to receive 3249 square rods of land in one square lot; what will be the length of one side of the field?

$$\dot{3}2\dot{4}9$$

First point the sum to distinguish the periods. I point it off into periods of two figures each, because the square of a number contains twice as many figures as the root, or one less than twice as many. If the left-hand period contain two figures, then the given sum contains twice as many figures as the root. The periods serve to show how many figures will be contained in the root. Having found the

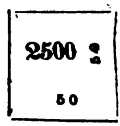
number of figures that will be contained in the root, find the greatest square of the left-hand period, and place its root at the right hand of the given sum.

3249|5

Now, because the root will contain two figures, the first figure (5) in the root is in the place of tens. This figure (5) represents one side of a square, which is found by multiplying 5 tens by 5 tens.

3249|5
25
———
749

When we have multiplied 5 tens by 5 tens, we find the product to be 250 tens, or 2500. If 50 r. represent one side of a square lot, that whole lot will contain 2500 rods, which may be represented by this figure, each of which sides may be supposed to be 50 rods long. We have now disposed 2500 rods of land into a square form; but there still remain to be disposed of, 749 rods more.

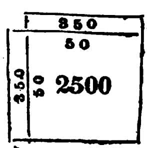


3249|5
25
———
749

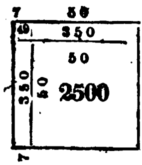
As we know the length of each side of the square, we know how long our additions must be. And in order to preserve the square form of the lot, we must make additions on two sides of the field, so as to lengthen all the boundary lines equally. Our next inquiry is, What must be the *width* of our additions? We have already shown, (XXI. SQUARE AND CUBIC MEASURE,) that when the *length* and the *square contents* of a surface are given, we can find the *width* by dividing the *square contents* by the length. The square rods to be added to the above lot, are 749. The length of additions is $50 \times 2 = 100$. Therefore, if we divide 749 by 100, the quotient will be the *width* of the additions.

3249|57
25
———
107|749
749
———

But because the 5 is in place of tens, and the cipher is omitted, therefore I must omit the cipher in the divisor, and one figure in the dividend. Now, if I make additions on two sides, the length of each addition will be 50 rods, its width 7, and its square contents 350 rods, according to the accompanying figure. The deficiency in the upper corner is 7 rods one way, and 7 rods the other; or each side of the deficiency is equal



to the width of the additions. Therefore, by placing the width of the addition at the right hand of the divisor, and multiplying it by itself, the square contents of this deficiency is found. This deficiency requires 49 rods, which complete the square form of the lot.



PROOF.—The whole length of one side of the lot multiplied by itself, will give the area, according to XXI. SQUARE AND CUBIC MEASURE. Or the parts of the lot may be added thus:

$$\begin{array}{r} 2500 \\ 350 \\ 350 \\ 49 \\ \hline 3249 \end{array}$$

EXAMPLES FOR PRACTICE.

1. What is the length of one side of a square field which contains 7056 rods? *Ans.* 84.

2. A general has 15625 soldiers; how many must he place in rank and file to form them into a square? *Ans.* 125.

3. There is a circular field which contains 1406,25 square feet; what must be the length of the side of a square field which will contain the same number of feet? *Ans.* 37,5.

The square of the longest side of a right-angled triangle is equal to the sum of the squares of the other two sides.

4. The top of a castle from the ground is 45 yards high, and is surrounded with a ditch 60 yards broad; how long must a line be to reach from the outside of the ditch to the top of the castle? *Ans.* 75 yards.

Circles are to each other as the squares of their diameters

5. There is a circle whose diameter is 4 inches; what is the diameter of a circle three times as large?

Ans. 6,928+

6. Two ships sail from the same port; one goes due north 45 leagues, and the other due west 76 leagues. How far are they apart? *Ans.* 88,32 leagues.

7. If a pipe 6 inches in diameter will conduct off a certain quantity of water in 4 hours, in what time will three pipes, each 4 inches in diameter, conduct off double the quantity? *Ans.* 6 hours.

8. If a pipe whose diameter is 1,5 inches, fill a cistern in 5 hours, in what time will a pipe whose diameter is 3,5 inches, fill the same? *Ans.* 55m. $6\frac{6}{49}$ sec.

XXVI. Cube Root.

When a number has been multiplied by itself *twice*, we obtain a product which is called a *cube*. When a number is considered in relation to such a product, it is called a *cube root*. To find the cube root of any number, therefore, is to find a number which, multiplied by itself *twice*, will produce the given cubic number.

TO FIND THE CUBE ROOT.

RULE.—*Beginning with the place of units, distinguish the sum into periods of three figures each, by placing a dot over units, thousands, and so on. If the sum contain decimals, place a dot over thousandths, millionths, and so on.*

Find, by trial, the greatest cube in the left-hand period, and place its root in the quotient. Subtract the cube thus found from the left-hand period, and to the remainder annex the second period, and call them the first dividend. Multiply the square of the root by 3, to this product add the product of the root multiplied by 3, call the amount the divisor, by which divide the first dividend, and place the quotient in the root. Multiply three times the square of the root (except the last figure in the root) by the last figure in

the root, and place the product under the first dividend. Multiply the square of the last quotient figure by the former figure, or figures, of the root, and this product multiply by 3, and its product place under the first dividend. Under all write the cube of the last quotient figure, and the amount of these three products subtract from the first dividend, and to the remainder bring down the third period, and call them the second dividend, with which proceed as before, until the whole root is found.

Note.—In squaring the root, it must be remembered that the value of the root must be expressed in a denomination corresponding with the period, the root of which you are finding. If you are finding the root of the second period from the right hand, reckon your root as tens; if that of the third period, call your root hundreds, omitting one cipher in the root for every period at the right hand of that period, the root of which you are seeking. Call the first right-hand period, reckoning from units, a unit period, the next left-hand period tens period, and so on.

1. What is the cube root of 1953125?

```
       1953125|125, Ans.
       1
      ----          10×10×3=300
   330|953             10×3= 30
      ----                  ---
       600                  330   first divisor.
       120
         8        120×120×3=43200
     ------           120×3=  360
 43560|225125              -----
      ------                43560  2d divisor.
       216000
         9000        5×5×120×3=9000
          125            5×5×5=125
      ------
       000000
```

When getting my first divisor, I am seeking the root of the second period, or *tens* period, therefore I call my root *tens*. If I call the root tens, I find it to be 10 tens=100, the actual value of the root. And the exact value of the root must always be preserved in the expression. I square this root (10 tens) to find the square contents of one addition to the cubic pile already formed in disposing of 1000000. 10×10=100, which denote the surface of one addition. This 100 is not one hundred *units*, nor one hundred *tens*, but 100 *hundreds*, or 10 *thousands*. This will be readily seen by taking the root as units, thus, 100, and squaring it.

$$100\times100=10000.$$

As there are to be additions on three sides of a cubic pile, in order to preserve its cubic form, therefore we wish to obtain the surface of *three* additions. This we do by multiplying the surface of one addition by 3, thus: 100×3 =300. If we divide the remaining contents of the sum by the surface of *all* the additions, the quotient would be the *thickness* of these additions. But it is impossible to ascertain the surfaces of all the additions before we get the next quotient figure, or the *thickness* of the additions. Owing to the imperfection of our divisors,—an imperfection which the nature of the subject will not allow us to remove,—the figure found by division to express the *thickness* of the addition, is not always the right one; and we are obliged to ascertain the correct figure, in some cases, by trial. But we can approximate a true divisor by ascertaining the surface of *several* additions, and thus determine, in most cases, what the *thickness* of the addition is, by division. Thus, 300 represent the surface of *three* additions. But there are *seven* additions, in all, to be made, before the cubic form of the second pile will be completed.

Six additions have the same *length*. Three of these additions have their surfaces determined; the other three are each 100 in length, or 10 *tens* in length. The length of all three are 10×3=30 tens. As the *width* of these three additions is the same as their *thickness*, and as their *thickness* is *not known*, therefore nothing further can be determined concerning these three additions, until their thickness is ascertained.

By adding the *length* of three additions to the *surface* of three additions, it is plain that we approximate more nearly the true divisor, than we should by taking the *surface* of three only. We see, also, why we do not multiply the whole divisor by the figure representing the thickness. 30 tens show the *length* of three additions, and should 30 be multiplied by 2 tens, the thickness of these additions, the product would not be the solid contents of these additions. Therefore we multiply the 300, representing the *surface* of three additions, by 2 tens, the thickness of these additions, and the product is the solid contents of these additions. 300 hundreds, multiplied by 2 tens, give 600 thousands. And as we are seeking the root of tens period, we omit the three ciphers, and subtract 600 from the tens period. The same result, however, would be obtained, had the *multiplication* been made in full. Thus, $30000 \times 20 = 600000$, which, subtracted from *units* and *tens* periods, would give the same remainder that we had before.

A more minute explanation of the rule will be given hereafter.

2. What is the cube root of 117649? *Ans.* 49.
3. What is the cube root of 83453453? *Ans.* 437.
4. What is the cube root of 15625? *Ans.* 25.
5. What is the cube root of 962966796875? *Ans.* 9875.
6. What is the cube root of 52734375? *Ans.* 375.
7. What is the cube root of 926859375? *Ans.* 975.
8. What is the cube root of 36926037? *Ans.* 333.
9. What is the cube root of 505752? *Ans.* 78.
10. What is the cube root of 14526784? *Ans.* 244.
11. What is the cube root of 13312053? *Ans.* 237.
12. What is the cube root of 75151448? *Ans.* 422.
13. What is the cube root of 398688256? *Ans.* 736.
14. What is the cube root of 54010152? *Ans.* 378.
15. What is the cube root of 13669062471? *Ans.* 2391.
16. What is the cube root of $\frac{27}{64}$? *Ans.* $\frac{3}{4}$.
17. What is the cube root of $\frac{216}{4096}$? *Ans.* $\frac{6}{16}$.

18. What is the cube root of $\frac{1331}{19683}$? *Ans.* $\frac{11}{27}$.
19. What is the cube root of $\frac{4913}{262144}$? *Ans.* $\frac{17}{64}$.
20. What is the cube root of $\frac{79507}{531441}$? *Ans.* $\frac{43}{81}$.
21. What is the cube root of $\frac{15625}{54872}$? *Ans.* $\frac{25}{38}$.
22. What is the cube root of $\frac{373248}{12326391}$? *Ans.* $\frac{72}{231}$.
23. What is the cube root of $\frac{83453453}{761048497}$? *Ans.* $\frac{437}{913}$.
24. What is the cube root of $\frac{29791}{74088}$? *Ans.* $\frac{31}{42}$.
25. What is the cube root of $\frac{6859}{132651}$? *Ans.* $\frac{19}{51}$.
26. What is the cube root of $\frac{,125}{1331}$? *Ans.* $\frac{,5}{11}$.
27. What is the cube root of $\frac{,008}{27}$? *Ans.* $\frac{,2}{3}$.
28. What is the cube root of $\frac{,216}{729}$? *Ans.* $\frac{,6}{9}=\frac{,2}{3}$.
29. What is the cube root of 1371700960631? *Ans.* 11111.
30. What is the cube root of 1030301? *Ans.* 101.
31. What is the cube root of 32,768? *Ans.* 3,2.
32. What is the cube root of 4,913? *Ans.* 1,7.
33. What is the cube root of $\frac{1}{8}$? *Ans.* $\frac{1}{2}$.
34. What is the cube root of $\frac{1}{2197}$? *Ans.* $\frac{1}{13}$.
35. What is the cube root of $\frac{4,913}{8}$? *Ans.* $\frac{1,7}{2}$.
36. What is the cube root of one thousand? *Ans.* 10.
37. What is the cube root of fifty sixes?

The following remark may be of some service to the student in extracting the cube root. If the last figure of the cube be 1, the last figure of the root will be 1;

if 2,	"	"	"	"	8;
if 3,	"	"	"	"	7;
if 4,	"	"	"	"	4;
if 5,	"	"	"	"	5;
if 6,	"	"	"	"	6;
if 7,	"	"	"	"	3;
if 8,	"	"	"	"	2;
if 9,	"	"	"	"	9.

When there are only three periods in the cube, only *one* divisor needs be found, because the last figure of the root may be known without dividing. And the process with the divisor found, needs be carried only so far as to ascertain

the *second* figure, when you may write the third figure in the root, and the work is done. Thus: What is the cube root of 1953125?

$$\dot{1}95\dot{3}12\dot{5}\,|\,125,\ \textit{Ans.}$$

1953125 | 125, *Ans.*	
1	$10\times10\times3=300$
—	$10\times3=\ \ 30$
330 | 953	——
	330

I find my second figure to be 2 tens, and as I know what the next figure will be, I stop the process here. And, universally, no divisor needs be found for the *unit period.* The same rule obtains in decimals, calling the right-hand period unit period, and so on, pointing off the *root*, according to the nature of the sum.

The student should now attend carefully to the following illustration and explanation of the rule. Suppose a man has 1953125 solid feet of brick, which he wishes to pile up in a cubical form; what must be the length of one side of this pile?

1953125 | 1
1
—

Having piled up 1000000 solid feet in pile No. 1,* there remain 953125 solid feet to be disposed of. As No. 1 is a cubic pile, when we commence enlarging it, we must pile brick on three sides of it. We know how long the sides are—they are 100 feet long. Our additions, then, must be 100 feet in length. But before we commence the work, we wish to know how *thick* our new laying of brick shall be. This we must ascertain by ciphering.

330 | 953125 | 2	$10\times10\times3=300$
	$10\times3=\ \ 30$
	——
	330

Square the root, (100, or 10 *tens*,) and we have the *sur-*

* The teacher should prepare blocks to represent the different piles of brick here referred to; thus the scholar will obtain a much more definite idea of the sum, than he could from drawings on the black board or cuts in the book.

face of one addition. Multiply the square of the root by 3, and the product is the surface of three additions. As the root is the length of each side of No. 1, multiply the root by 3, and the product is the length of three additions. The *surface* of three additions, and the length of three more added, are the divisor.

As the root is 100, the square of it is 10000. But as we are seeking the root of a ten's period, therefore we call the root 10 tens, omitting the cipher, because the right-hand or unit period is omitted for the present. For the same reason, multiply 10 tens by 3, to obtain the length of three additions. The quotient arising from dividing 953125 by 330 (thousands) is 2 (tens,) which represents the thickness of our additions made in pile No. 2. The three additions made in pile No. 2 are 20 feet thick, 100 feet long, and 100 feet wide.

The solid contents of these additions may be found according to XXI. SQUARE AND CUBIC MEASURE.

$100\times100\times20=200000$ ft. contents of 1st addition.
$100\times100\times20=200000$ ft. " 2d "
$100\times100\times20=200000$ ft. " 3d "

600000 feet, contents of the three additions.

Or thus: $100\times100\times20\times3=600000$.

But we see that No. 2 is not a cube, but that there are three deficiencies. The additions to be made in these corners are of the same length as the sides of pile No. 1 and the additions made in No. 2, that is, 100 feet. The thickness of the additions to be made in No. 2, is the same as that of the additions already made, that is, 20 feet. The width of the additions to be made in No. 2 is also 20 feet. We find the surface of one end of these additions by squaring the last quotient figure, 2 (tens.) We find the solid contents of one addition by multiplying the area of one end by the length, 100 feet. Thus, $20\times20\times100=40000$ feet, the solid contents of one addition. The contents of three are found by multiplying 40000 by 3, thus, $40000\times3=120000$ The additions last made are in No. 3; and the solid contents of the last three additions are 120000 solid

feet. We see, also, by inspecting No. 3, that it is not yet a perfect cube, but that there is a deficiency in the corner This deficiency will require a pile of brick 20 feet long, 20 feet wide, and 20 feet thick, to fill it. We find the contents of a pile that will fit by cubing the last quotient figure, 2 (tens;) thus, $20\times20\times20=8000$. By inspecting No. 4, we find that the last addition of 8000 feet completes the cubical pile.

We have now piled the following number of feet of brick: In No. 1, there were 1000000 feet. In No. 2, there were added 600000 feet more, which made the contents of pile No. 2, 1600000 feet. In No. 3, there were added 120000, which made the contents of No. 3, 1720000 feet. In No. 4, there were added 8000 feet, which made the contents of No. 4, 1728000 solid feet.

We have now a cubical pile, (No. 4;) but we have not disposed of all our brick. If the student is not tired of piling brick, we will now dispose of the remainder, 225125 solid feet. The student must remember that we are to make additions upon pile No. 4, the length of one of whose sides is 120 feet. We know the length and width of our additions, and wish to ascertain what their thickness will be. This we must ascertain, as before, by ciphering.

$$43560\,|\,\dot{2}2\dot{5}125\,|\,125 \qquad \begin{array}{r} 120\times120\times3=43200 \\ 120\times3=\quad 360 \\ \hline 43560 \end{array}$$

By dividing the remaining number of feet by the surface of three additions, and the length of three, we find their thickness to be 5 feet. The surface of one addition is 14400 feet, and the surface of three is three times as much as that of one. $14400\times3=43200$, the surface of these three additions, will be seen in No. 5.

The solid contents of these three additions may be found according to XXI. SQUARE AND CUBIC MEASURE. Thus, $120\times120\times5=72000$ feet, solid contents of one addition; $72000\times3=216000$ feet, the solid contents of three additions.

In order to complete the cubical form of No. 5, we must

make additions in the deficiencies. These additions will be 120 feet long, and 5 feet thick.

These additions are made in No. 6. The solid contents of one of these additions may be found thus: $120 \times 5 \times 5 = 3000$ feet. Three additions contain three times as many feet as one. $3000 \times 3 = 9000$ feet, solid contents of the three additions made in No. 6.

But the pile is not a cube. There must be an addition 5 feet wide, 5 feet thick, and 5 feet long. This addition is made in No. 7.

We have now made the following disposition of the brick:

In No. 1,	1000000	solid feet.
In No. 2,	600000	(added.)
In No. 3,	120000	"
In No. 4,	8000	"
In No. 5,	216000	"
In No. 6,	9000	"
In No. 7,	125	"
Total,	1953125	

In pile No. 7 there are 1953125 feet of brick, and it is in a cubical form, and one of its sides is 125 feet long.

Ans. 125 feet.

From the foregoing illustration of the principles of the cube root, the teacher will discover the propriety of referring frequently to XXI. SQUARE AND CUBIC MEASURE. The contents of the several additions should be exactly measured by the scholar, according to the rules of this section. Let this course be thoroughly attended to by the teacher, and by the aid of blocks the scholar will find little or no difficulty in fully comprehending all points relating to this rule.

No person can understand the nature of the operations performed in this rule, without an ocular illustration from the use of blocks. To aid the teacher and scholar in forming a set of blocks, and also in obtaining a more correct idea of the nature of the process required by the rule, reference has been made to each addition, according to the order in which it was made.

The mind of teacher and scholar cannot be too deeply impressed with the necessity of devoting particular attention to this subject, if a correct idea of it be desired.

EXAMPLES FOR PRACTICE.

1. If a ball, 3 inches in diameter, weigh 4 pounds, what will be the weight of a ball that is 6 inches in diameter? *Ans.* 32 lbs.

Spheres are to each other as the cubes of their diameters.

2. If the solid contents of a globe are 10648, what is the side of a cube of equal solidity? *Ans.* 22.

3. If a ball, weighing 4 pounds, be 3 inches in diameter, what will be the diameter of the same quality, weighing 32 pounds? *Ans.* 6 inches.

4. There is a cubical vessel whose side is 12 inches, and it is required to find the side of another vessel that is to contain three times as much? *Ans.* 17,306.

Cubes, and all similar solid bodies, are to each other as the cubes of their diameters.

5. There is a cellar dug, that contains a space of 1728 feet; what is the length of one side of the cellar? *Ans.* 12 feet.

6. There is a cubical piece of timber, that contains 103823 solid inches; what is the length of one side? *Ans.* 47 inches.

INVOLUTION.

Involution is finding any required power. Thus, to find the square of any number, multiply that number by itself.

Thus, $2\times2=4$, the second power of 2;
$4\times4=16$, the second power of 4;
$8\times8=64$, the second power of 8.

To find the cube, or third power, of any number, multiply that number by itself *twice*. Thus,

$2\times2=4\times2=8$, third power of 2;
$3\times3=9\times3=27$, third power of 3;
$4\times4=16\times4=64$, third power of 4;
$5\times5=25\times5=125$, third power of 5.

To find the fourth power of any number, multiply that number by itself three times.

The following table of *powers* may be useful for occasional inspection.

TABLE OF POWERS.*

1st P.	1	2	3	4	5	6	7	8	9
2d P.	1	4	9	16	25	36	49	64	81
3d P.	1	8	27	64	125	216	343	512	729
4th P.	1	16	81	256	625	1296	2401	4096	6561
5th P.	1	32	243	1024	3125	7776	16807	32768	59049
6th P.	1	64	729	4096	15625	46656	117649	262144	531441
7th P.	1	128	2187	16384	78125	279936	823543	2097152	4782969
8th P.	1	256	6561	65536	390625	1679616	5764801	16777216	43046721

EVOLUTION.

Evolution is finding the roots of powers. To distinguish *roots*, names may be employed to designate them, as in the case of powers. This sign $\surd$ denotes the square root. The square root may be called the *second root*, the cube root the *third root*, and so on.

The cube root may be denoted thus, $\surd^3$; the fourth root thus, $\surd^4$; fifth root thus, $\surd^5$; sixth root thus, $\surd^6$, and so on. Numbers, whose precise roots can be found, are called *rational*. Those, whose precise roots cannot be found, are called *irrational*, or *surd* numbers.

* *Names of Powers.*—The first power is the root; the 2d, square; the 3d, cube; the 4th, biquadrate; the 5th, sursolid; the 6th, square cube; the 7th, second sursolid; the 8th, biquadrate squared, &c.

SUNDRY RULES IN EVOLUTION.

TO EXTRACT THE BIQUADRATE OR FOURTH ROOT.

Rule.—*Extract the square root of the given number, and then extract the square root of the root so found, and the last root so found will be the fourth, or biquadrate root.*

What is the biquadrate root of 5308416? *Ans.* 48.

TO EXTRACT THE SURSOLID OR FIFTH ROOT.

Rule.—*Divide the given sum by five times the assumed root, whatever it may be, and to the quotient add one twentieth part of the fourth power of the same root.*

From the square root of this sum subtract one fourth part of the square of the assumed root. To the square root of the remainder add one half of the assumed root, and the sum is the root required, or an approximation to it.

This rule gives the root to *five* places, and generally to eight or nine places at the first process.

What is the fifth root of 281950621875? *Ans.* 195.

TO EXTRACT THE SIXTH ROOT.

Rule.—*Extract the square root of the given number, and then extract the cube root of that root.*

What is the sixth root of 191102976? *Ans.* 24.

TO EXTRACT THE EIGHTH ROOT.

Rule.—*Extract the square root of the given number continually, until you have three roots; the last of these is the root sought.*

What is the eighth root of 1785793904896? *Ans.* 34.

TO EXTRACT THE NINTH ROOT.

RULE.—*Extract the cube root of the given number, and you have the third power, whose cube root is the root sought.*

What is the ninth root of 5159780352? *Ans.* 12.

XXVII. Progression.

When unequal quantities occur, they may be considered in two different points of view: we may inquire, *how much* greater one quantity is than the other; or we may ask, *how many times* one quantity is greater than the other. The answer to the first question is found by *subtraction*, and the answer to the second by *division*. Although both results are ratios, yet mathematicians have distinguished them into the arbitrary divisions of ARITHMETICAL RATIO, and GEOMETRICAL RATIO. Hence, PROGRESSION is of two kinds—ARITHMETICAL and GEOMETRICAL.

ARITHMETICAL PROGRESSION.

When a series of numbers increases or decreases by the same quantity, it is called an *arithmetical progression*. When a series of numbers increases by the same quantity, it is called an *ascending* series. Thus, 1, 4, 7, 10, 13, 16, 19, 22, &c. are an *ascending* series of numbers. When a series of numbers decreases by the same number or quantity, it is called a *descending* series; thus, 22, 19, 16, 13, 10, 7, 4, 1. The number which expresses *how much* greater, or *how much* less, one term of the series is, than the next term in the same series, is called the *difference*.

Numbers may be employed to denote the number of terms in any series. These numbers are called *indices*.

Thus, *Indices*,	1,	2,	3,	4,	5,	6,	7.	
Arith. Progression,	3,	7,	11,	15,	19,	23,	27,	&c.

From these remarks, the propriety of the following rule will be seen.

THE FIRST TERM, THE COMMON DIFFERENCE, AND THE NUMBER OF TERMS BEING GIVEN, TO FIND THE LAST TERM.

RULE.—*Multiply the common difference by the number of terms less* 1, *and to the product add the first term.*

1. If the common difference be 4, the first term 3, and the number of terms 5, what is the last term?

We see that this sum may be performed at once by casting the eye upon the *indices* and *series* given above. In 5 terms, the common difference is taken 4 *times*, and the first term *once*. Therefore, $4\times4+3=19$, the last term.

2. If the first term be 3, the common difference 4, and the number of terms 7, what is the last term?

In 7 terms, the common difference is taken 6 *times*, and the first term *once*. Therefore, $6\times4+3=27$, last term.

3. If a man has $1 on simple interest, to what sum will it amount in 20 years?

The first term is $1, the common difference $,06, and the number of terms 21. *Ans.* $2,2.

The number of years is one less than the number of terms.*

THE FIRST TERM, THE LAST TERM, AND NUMBER OF TERMS GIVEN, TO FIND THE DIFFERENCE.

RULE.—*Subtract the first term from the last, divide the remainder by the number of terms less* 1, *and the quotient will be the difference.*

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* It will readily be seen that the *years* are one *less* than the number of terms, from the following:

Years,		1	2	3	4	5	6	7
Indices,	1	2	3	4	5	6	7	8
Arith. Prog.	$1,	1,06,	1,12,	1,18,	1,24,	1,30,	1,36,	1,42, &c.

One dollar is a *term* in the *series*, but the first year ranges over the *second term*, $1,06, the third year over the fourth term, and the seventh year over the eighth term.

1. If the first term be 3, the last term 27, and the number of terms 7, what is the difference?

By inspecting the *indices* and *series* before given, it will be found that, subtracting 3, the first term from 27 gives the remainder 24, which are composed of the *difference* taken 6 *times*. Therefore, $24\div 6=$ *Ans.* 4, *difference*.

2. If the first term is 4, the last term 39, and the number of terms 8, what is the difference? *Ans.* 5.

3. A man has 5 sons and 5 daughters, whose several ages differ alike; the youngest of the children was 3 years of age, and the oldest 48. What was the common difference of their ages? $48-3\div\overline{10-1}=5$, *Ans.*

4. A certain school consists of 19 scholars; the youngest is 3 years of age, and the oldest 39. What is the common difference of their ages? *Ans.* 2 years.

THE FIRST AND LAST TERMS, AND THE COMMON DIFFERENCE GIVEN, TO FIND THE NUMBER OF TERMS.

RULE.—*Divide the difference between the first and last terms by the common difference, and to the quotient add* 1.

1. If the first term is 3, the last term 27, and the common difference 4, what is the number of terms?

By inspecting the series before given, we shall find that after 3 are subtracted from 27, the remainder 24 is composed of the common difference taken 6 times. Therefore, $27-3\div 4+1=7$, number of terms. *Ans.* 7.

As 3, the first term, are subtracted from 27, and the remainder, 24, divided by 4, gives 6, the number of times which the common difference is taken, 1 must be added to 6, to bring into the computation the first term, 3.

2. If the first term is 3, the common difference 4, and the last term 35, what is the number of terms? *Ans.* 9.

3. If the first term is 4, the common difference 2, and the last term 22, what is the number of terms? *Ans.* 10.

4. If the first term is 4, the common difference 2, and the last term 14, what is the number of terms? *Ans.* 6.

5. If the first term is 5, the common difference 4, and the last term 45, what is the number of terms? *Ans.* 11.

6. If the first term is $3\frac{1}{3}$, the last term $7\frac{2}{3}$, and the common difference $1\frac{4}{9}$, what is the number of terms? *Ans.* 4.

7. If the first term is 2, the last term 6, and the common difference $1\frac{1}{3}$, what is the number of terms? *Ans.* 4.

THE FIRST AND LAST TERM, AND THE NUMBER OF TERMS GIVEN, TO FIND THE SUM OF ALL THE TERMS.

RULE.—*Multiply the sum of the first and last term by the number of terms, and half the product will be the sum of all the terms. Or, which is equally correct, multiply the sum of the first and last term by half the number of terms.*

1. If the first term is 2, the last term 29, and the number of terms 10, what is the sum of all the terms? *Ans.* 155.

ILLUSTRATION.

$$\begin{array}{r} 2+5+8+11+14+17+20+23+26+29 \\ 29+26+23+20+17+14+11+8+5+2 \\ \hline 31+31+31+31+31+31+31+31+31+31= \\ =310\div2=155. \end{array}$$

Or, which amounts to the same, $31\times10=310\div2=155$.

2. If the first term is 3, the last term 45, and the number of terms 22, what is the sum of all the terms? *Ans.* 528.

3. If the first term be 4, the common difference 3, and the number of terms 100, what is the last term? *Ans.* 301.

4. There are, in a certain triangular field, 41 rows of corn; the first row, in one corner, is a single hill; the second contains 3 hills, and so on, with a common difference of two; what is the number of hills in the last row?

Ans. 81 hills.

5. If 100 stones be placed in a straight line, at the distance of a yard from each other, how far must a person travel to bring them one by one to a box placed at the distance of a yard from the first stone? *Ans.* 5 mi. and 1300 yds.

GEOMETRICAL PROGRESSION.

Numbers, increasing or decreasing by a certain number of times, form a *geometrical progression*. Or, quantities are in geometrical progression when they increase by the same multiplier, or decrease by the same divisor.

The multiplier or divisor is called the *ratio*. When numbers increase by the same multiplier, they form an *ascending* series; when they decrease by the same divisor, they form a *descending* series.

THE RATIO, NUMBER OF TERMS, AND ONE EXTREME GIVEN, TO FIND THE OTHER EXTREME.

RULE.—*When the series is ascending, multiply the given extreme by such power of the ratio, whose index is equal to the number of terms less* 1, *and the product will be the other extreme.*

When the series is descending, divide the given extreme by such power of the ratio, whose index is equal to the number of terms less 1, *and the quotient will be the other extreme.*

The reason of the rule is evident from the manner in which a geometrical progression is formed. Let 2, 4, 8, 16, 32, &c. be the series whose ratio is 2. The *second* term is formed by multiplying the first by the ratio; the *third* term by multiplying the second by the ratio, and so on. The series may therefore be written thus: $2, 2\times2, 2\times2\times2, 2\times2\times2\times2, 2\times2\times2\times2\times2$, &c. Any term after the first is evidently that power of the ratio whose index is one less than the number of the term multiplied by the first term. Thus, the third term is 2×2^2; the fourth term is 2×2^3; and the eighth term would be 2×2^7, and so on. In an ascending series, therefore, multiply the first term by that power of the ratio, whose index is one less than the number of the term sought, and the product is the term sought. In a descending series, as, 243, 81, 27, 9, 3, 1, whose ratio is 3, and which is also $1\times3^5, 1\times3^4, 1\times3^3, 1\times3^2, 1\times3^1, 1$, the last term, $1=\frac{1\times3^5}{3^5}$.

1. If the first term be 4, and the number of terms 9, what is the last term?

Indices,* 1, 2, 3, 4+4=8
4, 16, 64, 256×256=65536=power of the ratio, whose exponent is less by 1 than the number of terms.

65536=8th power of the ratio.
65536×4=262144=the last term.

2. If the last term be 262144, the ratio 4, and the number of terms 9, what is the first term? *Ans.* 4.

THE FIRST TERM AND THE RATIO GIVEN, TO FIND ANY OTHER REQUIRED TERM.

RULE.—*When the indices begin with a unit, write down a few of the leading terms of the series, and place their indices over them. Add those indices, the sum of which will make up the index to the required term. Multiply the terms of the geometrical series belonging to those indices together, and the product will be the term sought.*

When the indices begin with a cipher, write down a few of the leading terms of the series, as before, and place their indices over them. Add together the most convenient indices to make an index, less by 1 than the number expressing the

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* When the *first* term of the series and the *ratio* are *equal*, the *indices* must begin with a unit, and, in this case, the product of any two terms is equal to that term signified by the *sum* of the *indices*. Thus, $\begin{cases} 1, 2, 3, 4, 5, 6, \&c. \textit{ indices.} \\ 2, 4, 8, 16, 32, 64, \&c. \textit{ geometrical series.} \end{cases}$

6+6=12, the index of the twelfth term.
64×64=4096, the twelfth term.

But when the first term of the series and the ratio are different, the indices must begin with a cipher, and the sum of the indices, made choice of, must be 1 less than the number of terms given in the question. Thus,

$\begin{cases} 0, 1, 2, 3, 4, 5, 6, \&c., \textit{ indices.} \\ 1, 3, 9, 27, 81, 243, 729, \&c., \textit{ geometrical series.} \end{cases}$

6+5=11, the index of the twelfth term.
729×243=177147, the 12th term.

place of the term sought. Multiply the terms of the geometrical series together belonging to these indices, and make the product a dividend. Raise the first term to a power, the index of which is one less than the number of terms multiplied, and make the result a divisor, by which divide the dividend, and the quotient will be that term, beyond the first, signified by the sum of those indices, or the term sought.

1. If the first term be 2, and the ratio 2, what is the 13th term?

$$1,\ 2,\ 3,\ 4,\ 5+5+3=13$$
$$2,\ 4,\ 8,\ 16,\ 32\times32\times\ 8=8192, \textit{ Ans.}$$

2. A merchant wishing to purchase a cargo of horses for the West Indies, a jockey told him he would take all the trouble and expense upon himself, of collecting and purchasing 30 horses for the voyage, if he would give him what the last horse would come to, selling them at the rate of two farthings for the first, four for the second, eight for the third, &c. What was the cost of the last horse? and what was the average price of the horses?

Ans. £1118481 1 s. 4 d., price of the last horse.
£37282 14 s. 0½ d., average price of the horses.

3. If the first is 5, and the ratio 3, what is the 7th term?
Ans. 3645.

4. A man bought 20 cows, paying 2 farthings for the first, 10 for the second, and so on, in a five-fold ratio. What was the price of the last cow?
Ans. £39736429850 5 s. 2½ d.

THE FIRST TERM, THE RATIO, AND NUMBER OF TERMS GIVEN, TO FIND THE SUM OF THE SERIES.

RULE.—*Raise the ratio to a power, the index of which shall be equal to the number of terms, from which subtract 1; divide the remainder by the ratio less 1, and the quotient, multiplied by the first term, will give the sum of the series.*

1. If the first term is 5, the ratio 3, and the number of terms 7, what is the sum of the series?

Ratio, $3^7=2187=$ 7th power of the ratio.
Subtract 1

Divide by the ratio less 1, $3-1=2\,|\,2186$

1093, quotient.

Multiply by the first term, 5

Sum of the series, 5465

Or, $\frac{3^7-1}{3-1}\times5=5465$, *Ans.*

2. What debt can be discharged in a year, by paying 1 cent the first month, 10 cents the second, and so on, increasing in a ten-fold proportion each month?
Ans. \$1111111111,11.

3. What is the sum of the infinite series $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, &c.?
Ans. $\frac{1}{3}$.

4. What is the sum of the infinite series ,1, 01, 001, &c.?
Ans. $\frac{1}{9}$.

THE EXTREMES AND THE RATIO GIVEN, TO FIND THE SUM OF THE SERIES.

RULE.—*Divide the difference of the extremes by the ratio less* 1, *add the greater extreme to the quotient, and the result will be the sum of all the terms.*

1. A gentleman, whose daughter was married on a new-year's day, gave her a guinea, promising to triple it on the first day of each month in the year; to what did her portion amount? *Ans.* 265720.

2. Suppose a ball to be put in motion by a force which impels it 10 rods the first minute, 8 the second, and so on, decreasing by a ratio of 1,25 each minute to infinity; what space would it move through? *Ans.* 50 rods.

3. What is the value of ,999, &c. to infinity? *Ans.* 1.

XXVIII. Position.

Position is a method of performing such questions as cannot be resolved by the common direct rules, and is of two kinds, called *single* and *double.*

SINGLE POSITION.

SINGLE POSITION TEACHES TO RESOLVE THOSE QUESTIONS WHOSE RESULTS ARE PROPORTIONAL TO THEIR SUPPOSITIONS.

Rule.—*Take any number, and perform the same operations with it, as are described to be performed in the question. Then say, as the result of the operation is to the position, so is the result in the question to the number required.*

1. A's age is double that of B, and B's is triple that of C, and the sum of all their ages is 140; what is each person's age?

Suppose A's age to be 60
Then will B's $= \frac{60}{2} = 30$
And C's $= \frac{30}{3} = 10$

100

As 100 : 60 : : 140 : $=84=$ A's age.
Consequently $\frac{84}{2}=42=$ B's "
And $\frac{42}{3}=14=$ C's "

140

2. A certain sum of money is to be divided between four persons in such a manner, that the first shall have $\frac{1}{3}$ of it, the second $\frac{1}{4}$, the third $\frac{1}{6}$, and the fourth the remainder, which is £28; what is the sum? *Ans.* £112.

3. A person, after spending $\frac{1}{3}$ and $\frac{1}{4}$ of his money, had $60 left; what sum had he at first? *Ans.* $144.

4. What number is that which, being increased by $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of itself, the sum shall be 125? *Ans.* 60.

5. A person bought a chaise, horse and harness for £60; the horse came to twice the price of the harness, and the chaise to twice the price of the horse and harness; what did he give for each?

Ans. £13 6 s. 8 d. for the horse, £6 13 s. 4 d. for the harness, and £40 for the chaise.

6. A vessel has three cocks, A, B, and C; A can fill it in 1 hour, B in 2, and C in 3; in what time will they all fill it together? *Ans.* $\frac{6}{11}$ hour.

DOUBLE POSITION.

DOUBLE POSITION TEACHES TO RESOLVE QUESTIONS BY MAKING TWO SUPPOSITIONS OF FALSE NUMBERS.

RULE.—*Take any two convenient numbers, and proceed with each according to the conditions of the question. Find how much the results are different from the result in the question. Multiply each of the errors by the contrary supposition, and find the sum or difference of the product. If the errors be alike, divide the difference of the products by the difference of the errors, and the quotient will be the answer. If the errors be unlike, divide the sum of the products by the sum of the errors, and the quotient will be the answer.*

Note.—The errors are said to be *alike*, when they are both too great, or both too little; and *unlike*, when one is too great, and the other too little.

1. A lady bought tabby at 4 s. a yard, and Persian at 2 s. a yard; the whole number of yards she bought was eight, and the whole price 20 s.; how many yards had she of each sort?

Suppose 4 yards of tabby, value 16 s.
Then she must have 4 yards of Persian, value 8

Sum of their values 24

So that the first error is + 4

Again, suppose she had 3 yards of tabby at 12 s.
Then she must have 5 yards of Persian at 10

Sum of their values 22

So that the second error is + 2

Then, $4-2=2$=difference of the errors.

Also, $4\times2=8$=product of the first supposition and second error.

And $3\times4=12$=product of the second supposition by the first error.

And $12-8=4$=their difference.

Whence $4\div2=2$=yards of tabby, } *Ans.*
And $8-2=6$=yards of Persian, }

2. Two persons, A and B, have both the same income; A saves $\frac{1}{5}$ of his yearly; but B, by spending $50 a year more than A, at the end of four years finds himself $100 in debt; what is their income, and what do they spend a year?

Ans. Their income is $125; A spends $100, and B $150.

3. Two persons, A and B, lay out equal sums of money in trade; A gains $126, and B loses $87, and A's money is now double that of B; what did each lay out? *Ans.* $300.

4. A laborer was hired for 40 days on this condition, that he should receive 20 d. for every day he wrought, and forfeit 10 d. for every day he was idle: now he received at last £2 1 s. 8 d. How many days did he work, and how many was he idle? *Ans.* He wrought 30 days, and was idle 10.

5. A gentleman has two horses of considerable value, and a saddle worth £50; now, if the saddle be put on the back of the first horse, it will make his value double that of the second; but if it be put on the back of the second, it will make his value triple that of the first; what is the value of each horse? *Ans.* One £30, and the other £40.

6. There is a fish whose head is 9 inches long, and his tail is as long as his head and half as long as his body, and his body is as long as his tail and his head; what is the whole length of the fish? *Ans.* 6 feet.

XXIX. Mensuration.

MENSURATION COMPREHENDS THE ADMEASUREMENT OF SURFACES, AND THE SOLID CONTENTS OF BODIES.

TO FIND THE AREA OF ANY PARALLELOGRAM, WHETHER IT BE A SQUARE, A RECTANGLE, A RHOMBUS, OR A RHOMBOID.

RULE.—*Multiply the length by the perpendicular breadth or height, and the product will be the area.*

What is the area of a rhomboid, the length being 7, and the perpendicular being 4? *Ans.* 28.

TO FIND THE AREA OF A TRIANGLE.

RULE.—*Multiply the base by the perpendicular height, and take half the product for the area.*

What is the area of a triangle, the base being 9, and the perpendicular 8½? *Ans.* 38¼.

NAMES OF SEVERAL POLYGONS.

No. of sides.		*No. of sides.*	
5,	Pentagon,	9,	Nonagon,
6,	Hexagon,	10,	Decagon,
7,	Heptagon,	11.	Undecagon,
8,	Octagon,	12,	Dodecagon.

TO MEASURE ANY REGULAR POLYGON.

RULE.—*Multiply the length of one of the sides by the number of sides; then multiply this product by the half of a perpendicular let fall from the centre of the figure to the middle of one of the sides, and the product will be the area of the polygon.*

1. What is the area of a regular pentagon, each of its sides being 95, and its perpendicular 65,36? *Ans.* 15523.

2. What is the area of a regular nonagon, each of its sides being 5, and its perpendicular 6,8686935?

Ans. 154,54+.

THE DIAMETER OF A CIRCLE BEING GIVEN, TO FIND THE CIRCUMFERENCE.

RULE.—*As* 7 *are to* 22, *so is the diameter of a circle to the circumference. Or, as* 113 *are to* 355, *or* 1 *is to* 3,14159, *so is the diameter to the circumference.*

1. What is the circumference of a circle, whose diameter is 12? *Ans.* 37,69908.

2. What is the circumference of a circle, its diameter being 6? *Ans.* 18,8496.

THE CIRCUMFERENCE OF A CIRCLE BEING GIVEN, TO FIND THE DIAMETER.

RULE.—*As* 22 *are to* 7; *or* 355 *to* 113; *or as* 1 *is to* ,31831, *so is the circumference of a circle to its diameter.*

THE DIAMETER OF A CIRCLE BEING GIVEN, TO FIND ITS AREA.

RULE.—*Multiply the square of the diameter by* ,7854.

1. What is the area of a circle, the diameter of which is 12? *Ans.* 113,0976.

2. What is the area of a circle, the diameter being 9?

Ans. 63,6174.

THE CIRCUMFERENCE OF A CIRCLE BEING GIVEN, TO FIND THE AREA.

RULE.—*Multiply the square of the circumference by* ,07958.

1. What is the area of a circle, the circumference of which is 37,7? *Ans.* 113,1062582.

2. What is the area of a circle, the circumference of which is 28,27? *Ans.* 63,599.

THE AREA OF A CIRCLE BEING GIVEN, TO FIND THE DIAMETER.

RULE.—*Multiply the given area by* 1,2732, *and the product will be the square of the diameter; extract the square of this product, and you will have the diameter.*

What is the diameter of a circle, the area of which is 113,09? *Ans.* 11,999.

THE AREA OF A CIRCLE BEING GIVEN, TO FIND THE CIRCUMFERENCE.

RULE.—*Multiply the given area by* 12,566, *and extract the square root of the product, and the root will be the circumference.*

What is the area of a circle, the circumference of which is 113,03? *Ans.* 37,68.

THE SIDE OF A SQUARE BEING GIVEN, TO FIND THE DIAMETER OF A CIRCLE EQUAL TO THE SQUARE THE SIDE OF WHICH IS GIVEN.

RULE.—*Multiply the given side by* 1,128, *and the product will be the diameter of a circle, the area of which is equal to the area of the given square.*

What is the diameter of a circle equal to a square, one side of which is 10,635? *Ans.* 11,99628.

What is the diameter of a circle equal to a square, one side of which is 83,33⅓? *Ans.* 94.

TO FIND THE AREA OF AN ELLIPSIS, OR OVAL.

RULE.—*Multiply the longest diameter by the shortest; then multiply the product by the decimal* ,7854.

What is the area of an oval, its diameters being 7 and 5? *Ans.* 27,489.

TO FIND THE SURFACE OF A PRISM, OR CYLINDER.

RULE.—*Multiply the perimeter of one end of the prism by the length of the solid, and the product will be the surface of all its sides. To which add also the area of the two ends of the prism.*

What is the surface of the sides of an equilateral triangular prism, its length being 9, and side 3? *Ans.* 81.

TO FIND THE SURFACE OF A PYRAMID, OR CONE.

RULE.—*Multiply the perimeter of the base by the slant height or length of the side, and half the product will be the surface of the sides, or the sum of the areas of all the triangles which form it. To which add the area of the end or base.*

What is the surface of a pyramid, its slant height being 20, and the perimeter of its base 15? *Ans.* 150.

TO FIND THE SURFACE OF THE FRUSTUM OF A PYRAMID.

RULE.—*Add the perimeters of the two ends, and multiply their sum by the slant height, taking half the product for the answer.*

What is the surface of the frustum of a pyramid, the slant height being 12, the diameters 8 and 6?

Ans. 263,88.

TO MEASURE A CYLINDER.

RULE.—*The diameter of the base being given, find the area of the end; then multiply the area of the base by the length.*

What are the solid contents of a cylinder, 1 foot, 9 inches in diameter, and 12 feet, 6 inches long? *Ans.* 30,[illegible]

TO MEASURE A PRISM.

Rule.—*Find the surface of one end, and multiply that by the length.*

What are the contents of a prism, each of its sides being 10 inches, and the length 12 feet? *Ans.* 3,6.

TO MEASURE A PYRAMID.

Rule.—*Find the surface of the base, and multiply that by one third of the height.*

What are the solid contents of a triangular pyramid, each of its sides being 13, and the height 48? *Ans.* 1144.

TO MEASURE A SPHERE, OR GLOBE.

Rule.—*Multiply the cube of the diameter by ,5236.*

What are the solid contents of a globe, its diameter being 4,5? *Ans.* 47,71305.

XXX. Calculation of Speeds.

TO FIND THE SPEED OF ANY GIVEN SHAFT.

Rule.—*Begin at the water wheel, and trace out all the driving and driven wheels separately, from the first driver to the last driven on the first end of the given shaft. Multiply the number of teeth in all the driving wheels together, and the product by the revolutions of the water wheel per minute; then multiply all the driven wheels together, and divide the product of the former by the product of the latter; the quotient will express the revolutions per minute of the given shaft.*

There is a water wheel which performs 24 revolutions per minute. Connected with this water wheel are 2 driving wheels, and 3 driven wheels; the first driving or water wheel has 90 teeth, the second 54, and the third 48. The first driven wheel has 45, the second 36, and the third 36. How many revolutions per minute has the last driven wheel?

OPERATION.

1st driving or water wheel,........90 teeth.	1st driven wheel,.45 teeth.
2d driving wheel,.54	2d.............36
360	270
450	135
4860	1620
3d driving wheel,.48	3d driven wheel,..36
38880	9720
19440	4860
233280	58320
Revolutions per minute of wat. wh. 24	
933120	
466560	
5598720	

5598720÷58320=96 revolutions of the last driven wheel per minute. *Ans.* 96.

TO FIND THE REVOLUTIONS PER MINUTE OF THE LAST DRIVEN, WHEN BANDS OR BELTS ARE USED.

RULE.—*Trace out all the driving and driven pulleys or drums, from the main shaft to the one the revolutions of which are sought. Multiply the diameters of all the driving pulleys or drums together, and the product by the revolutions per minute of the main driving shaft; then multiply the diameters of all the driven pulleys or drums together, and*

divide the product of the former by the product of the latter; the quotient will express the revolutions per minute of the last driven shaft, or wheel.

Suppose the diameter of the driving drum on a main shaft to be 18 inches, and driving a changing or speed pulley of 16 inches; on the same shaft with which there is another driving drum of 18 inches, driving a pulley on the end of the cylinder, or frame shaft, of 16 inches diameter. If the speed of the main shaft is 108 revolutions per minute, what is the number of revolutions per minute of the last driven shaft, or wheel?

Speed of main shaft....108		Diam. of 1st driv. pulley.16	
Diam. of drum or shaft..18 in.		" 2d do.....16	
	864		96
	108		16
	1944		256
Diam. of 2d driving drum.18			
	15552		
	1944		

34992÷256=136,6875, the revolutions per minute of the last driven shaft.

Ans. 136,6875=136$\frac{11}{16}$.

It may be laid down as a general rule, that all these calculations of speeds proceed upon one uniform principle, viz. *Separate all the driving wheels or pulleys from the driven; and if wheels, multiply the number of teeth; but if pulleys or drums, multiply the diameters of all the drivers together, and the product by the speed given: then multiply all the driven together, and divide the product of the former by the product of the latter, and the quotient will be the speed sought.*

There is a water wheel, the velocity of which is 10 revolutions per minute. To this wheel is attached a driving wheel, containing 200 teeth. This wheel (of 200 teeth) drives a pinion (on the first driven shaft) of 40 teeth, and

on this shaft is a pulley or drum, of 48 inches diameter. This pulley drives, by means of a belt or band, a pulley of 24 inches diameter, which is on the main shaft, or drum. On this shaft there is a pulley of 18 inches diameter. The pulley of 18 in. diameter drives, by means of a belt or band, a pulley of 9 inches diameter. On the same shaft to which the pulley of 9 in. diameter is attached, is a pulley of 20 in. diameter. This pulley drives, by means of a belt or band, a pulley of 2 in. diameter, on a lathe spindle. Required the revolutions per minute of the lathe spindle.

Ans. 2000 revolutions per minute.

MISCELLANEOUS EXAMPLES.

1. At 2 bu. and 6 qts. of ashes per pound, how many bushels of ashes will pay for 11 pounds and 5 oz. of tea?

Ans. 24,74609375=24 bu. 23 qts. 7 gills.

2. At 12$\frac{1}{2}$ yards per cord for wood, how much cloth would be required to pay for 17 cords and 7$\frac{1}{2}$ cord feet of wood? *Ans.* 224,21875=224$\frac{7}{32}$ yds.

3. At 2 bu. and 12 qts. of wheat per barrel of soap, how many bushels of wheat would be required to pay for 79,26 barrels of soap? *Ans.* 188,2425=188 bu. 7,84 qts.

4. At one cord and two cord feet of wood per yard of broadcloth, how much wood must be given in exchange for 11$\frac{11}{16}$ yds.?

Ans. 14,609375=14 cords, 4 cord feet, 14 cubic feet.

5. At 2$\frac{1}{8}$ yards of cotton cloth per pound of butter, how much cloth would pay for 5 pounds and 3 ounces of butter?

Ans. 11,0234375=11 yds. and $\frac{6,75}{8}$ of an inch.

6. If a man turn 2 feet and 6 inches of iron in one hour, how much would he turn in 2 hours and 3 minutes?

Multiply 2 h. and 3 m. by 2 ft. and 6 in.

Ans. 5,125=5 feet and 1$\frac{1}{2}$ inch.

7. What would be the interest on 11 bu. and 11 qts. of potatoes for one year, at 1 quart of corn per bushel?

Ans. ,3544921875 of a bushel=11 qts. 2$\frac{3}{4}$ gills.

8. What would be the interest on 19 pounds and 15 oz. of butter for one year, at 1 pint of peas per pound?
Ans. ,3115234375 of a bushel=1 pk. 1 qt. 1 pt. $3\frac{3}{4}$ gills.

9. What would be the interest on 9 barrels and 13 galls. of cider for 2 years, 5 m. and 11 days, at 6 qts. of soap per barrel of cider per year? *Ans.* $1,07902425130208\frac{1}{3}$ of a bushel=1 bar. 2 galls. 2 qts. $,92\frac{1}{12}$ gills.

10. What would be the interest on 39 bu. 31 qts. of corn for 3 years, 7 months, and 17 days, at 13 ounces of cheese per bushel per year? *Ans.* $117,900873480902\frac{7}{8}$ pounds.

11. A gentleman, being asked how much money he had, replied, If I had as much more as I now have, $\frac{3}{4}$ as much, $\frac{1}{2}$ as much, $\frac{1}{4}$ as much, and $\frac{1}{8}$ as much, I should then have \$1305; what was the amount of money that he had?
Ans. \$360.

12. A, B, C, and D spent at a reckoning 105 s.; but being intoxicated, agreed that A should pay $\frac{2}{3}$ of this sum, B $\frac{1}{2}$, C $\frac{1}{3}$, and D $\frac{1}{4}$; what was each man's bill?

$\frac{2}{3}=,66\frac{2}{3}$	$105\div1,75=60\times\frac{2}{3}=40$ s.	A's share.
$\frac{1}{2}=,5$	$105\div1,75=60\times\frac{1}{2}=30$ s.	B's "
$\frac{1}{3}=,33\frac{1}{3}$	$105\div1,75=60\times\frac{1}{3}=20$ s.	C's "
$\frac{1}{4}=,25$	$105\div1,75=60\times\frac{1}{4}=15$ s.	D's "
1,75		

13. Divide \$2400 between A, B, C, D, E, F, G, and H, giving to A $\frac{1}{3}$, B $\frac{1}{4}$, C $\frac{1}{5}$, D $\frac{1}{6}$, E $\frac{1}{8}$, F $\frac{3}{5}$, G $\frac{9}{20}$, and H $\frac{3}{8}$.
Ans. A's, \$320; B's, \$240; C's, \$192; D's, \$160; E's, \$120; F's, \$576; G's, \$432; H's, \$360.

14. Divide \$92,15 among four men, giving them in the following proportion:—to A $\frac{1}{3}$, B $\frac{1}{4}$, C $\frac{1}{5}$, D $\frac{1}{6}$.
Ans. A's, $\$32,33\frac{1}{3}$; B's, \$24,25; C's, \$19,4; D's, $\$16,16\frac{2}{3}$.

To find the price of any thing by the ton, a ton being the unit. The point will be at the left of hundreds.

Multiply the whole number of pounds by one half the price. Or, divide the number of pounds by 2000, and the quotient will be tons and the decimal parts, which multiply by the whole price.

15. What will 1978 pounds of hay come to, at \$12 per ton? $1,978\times6=11,868$, *Ans.*
Or, $1978\div2000\times12=11,868$, *Ans.*

16. What will 5 tons, 1727 pounds of steel cost, at 200 per ton? *Ans.* 1172,7.

17. What will 9 tons, 97 pounds, and 11 ounces of hay cost, at \$9,375 per ton? *Ans.* 84,83291015625.

18. At $16\frac{2}{3}$ acres of country land per acre of village land, how much country land would pay for $\frac{11}{16}$ of an acre of village land? *Ans.* $11,458\frac{1}{3}$=11 acres, 1 r. $33\frac{1}{3}$ rods.

19. A gentleman, having some change in his pocket, accidentally lost $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{3}{8}$, and $\frac{1}{20}$ of a dollar; how much money did he lose? *Ans.* \$2.

20. Multiply 37865 by $25166\frac{2}{3}$.

```
4|6|37865          166⅔=⅙ of 1000
    25166⅔         25000 =¼ of 10000
    ------
     6310833⅓..product by 166⅔
    946625......product by 25000
    ---------
Ans. 952935833⅓
```

21. Multiply 528762 by 125250.

```
8|4|528762          250=¼ of 1000
    125250       125000=⅛ of 1000000
    ------
    132190500..product by 250
   66095250.....product by 125000
   -----------
Ans. 66227440500
```

22. Multiply 678537 by 875625.

```
8|678537            625=⅝ of 1000
  --------
    84817125..product by 125=⅛ of 1000
      7   5
  ---------
   424085625..product by 625=⅝ of 1000
593719875.....product by 875000=⅞ of 1000000
------------
594143960625..product by 875625, Ans.
```

23. If a man travel $3\frac{4}{5}$ miles in one hour, how far would he travel in 5 hours and 45 minutes?

Multiply $3\frac{4}{5}$ miles by 5 h. and 45 m.

Ans. 21,85, or $21\frac{17}{20}$ miles.

24. What is the difference between a floor 20 feet square, and two others, each 10 feet square? *Ans.* 200 feet.

25. Four men bought a grindstone of 60 inches in diameter: how much of its diameter must each grind off, to have an equal share of the stone, if one first grind his share, and then another, till the stone is ground away, making no allowance for the eye of the stone?

Ans. { 1st share, 8,0385. 2d share, 9,5351. 3d share, 12,4264. 4th share, 30.

26. Suppose a man has three horses, and his neighbor will give five horses for one of his; how many horses will be required to pay for the three horses?

Multiply 5 horses by 3 horses. *Ans.* 15 horses.

27. Suppose 37 colored boys were sold for 125 sheep each, how many sheep would pay for the boys?

Multiply 37 boys by 125 sheep. *Ans.* 4625 sheep.

TO MEASURE CASKS.

Multiply the square of the mean diameter by the length; multiply this product by 34 for wine, or by 28 for beer, and, pointing off four decimals, the product will be the contents in gallons and decimals of a gallon.

The mean diameter of a cask may be found by adding two thirds, or, if the staves be but little curving, six tenths, of the difference between the head and bung diameters to the head diameter.

28. How many wine gallons in a cask, the bung diameter of which is 36 inches, head diameter 27 inches, and length 45 inches? *Ans.* 166,6170.

29. An ignorant fop wanting to purchase an elegant house, a facetious gentleman told him he had one which he would sell him on these moderate terms, viz. that he should

give him a cent for the first door, 2 cents for the second, 4 cents for the third, and so on, doubling at every door, which were 36 in all. It is a bargain, cried the simpleton, and here is a guinea to bind it. What did the house cost him?

Ans. $687184767,35.

METHOD OF DIVIDING ESTATES IN CERTAIN CASES.

RULE.—*Ascertain the interest on $100 for the time each heir's share is to run before he arrives to the age of 21, to which add the $100; add the several amounts of principal and interest, thus ascertained, together, and by their sum divide the amount of the estate, and the quotient will give the present value of $100. Then multiply each of said several amounts by the quotient, and the products will be the respective shares of each in inverted order.*

Or, *Multiply the amount of the estate by each of said several amounts separately, and divide the respective products by the sum of said amounts, and the quotient will be the share of each in direct order.*

30. A dies, leaving 7 heirs, and an estate to the value of $12687,50 to be so divided among them, that on each arriving at the age of 21, their shares, at 6 per cent interest, shall all be equal. The ages of the heirs are as follows:— A, 19 years, 9 months; B, 17 years, 3 months; C, 15 years, 1,5 months; D, 13 years, 6 months; E, 11 years, 4,5 m.; F, 9 years, 7,5 months; G, 7 years, 10,5 months; what is the present value of the share of each?

Yrs. Ages. Time. Rate on $100. Am't.
A 21—19,75=1,25×6+100=107,5÷8=$1343,75, G's.
B 21—17,25=3,75..........122,5.....$1531,25, F's.
C 21—15,125=5,875........135,25....$1690,625, E's.
D 21—13,5 = 7,5...........145, ...$1812,5, D's.
E 21—13,375—9,625........157,75....$1971,875, C's.
F 21— 9,625=11,375.......168,25....$2103,125, B's.
G 21— 7,875=13,125.......178,75....$2234,375, A's.

1015,00 | 12687,5 | 12,5

Then 12,5=⅛ of 100. The multiplication may be performed by division. See *Multiplication*, page 81.

32. A man dies, leaving two sons, 14 and 18 years of age, and an estate of $1000, to be so divided between them that, at 21, their shares on interest at 6 per cent, shall be equal. What would be the present share of each?

Ans. The elder, \$546,15$\frac{5}{13}$; the younger, \$453,84$\frac{8}{13}$.

33. An estate of $159600 is left to six heirs, to be divided among them as in the preceding examples, their ages being as follows: A, 18 years, 7 months, 24 days; B, 15 years, 3 m. 18 d.; C, 13 years, 11 m. 3 d.; D, 9 years, 11 months, 12 d.; E, 6 years, 1 m. 6 d.; F, 2 years, 5 m. 21 d. What is the present value of each share?

Ans.
- A's share, \$35191,66$\frac{2}{3}$.
- B's " \$31566,66$\frac{2}{3}$.
- C's " \$27716,66$\frac{2}{3}$.
- D's " \$23741,66$\frac{2}{3}$.
- E's " \$22366,66$\frac{2}{3}$.
- F's " \$19016,66$\frac{2}{3}$.

34. An estate of $6984 is left to four heirs, to be divided among them as in the preceding examples. Their ages were as follows: A, 9 years, 11 months, 7 days; B, 7 years, 1 month, 9 days; C, 3 years, 3 months, 17 days; D, 1 year, 7 months, 27 days. What is the share of each now, and what the amount of each share at 21?

Present shares.

Ans.
- A, $1944,45.
- B, $1855,95.
- C, $1650,15.
- D, $1497,45.

Share at 21, $3235,240725.

Note.—To find any other rate per cent, use the figure representing the rate, as 6 is used for 6 per cent.

METHOD OF FINDING UNEQUAL SHARES, TO BE INCREASED ONE ABOVE ANOTHER IN A GIVEN RATIO.

RULE.—*Take from the amount to be divided the sum of the excesses over the least share; then divide the remainder by the number of shares; the quotient will show the least share. The excess to be given to each, being added to the quotient, will give the share of each.*

35. Divide \$600 among five men, so as to give B \$2 more than A, C \$2 more than B, D \$2 more than C, E \$2 more than D. What is the share of each?

A 0	\$600	Ans.	A, \$116.
B 2	20		B, \$118.
C 4	——		C, \$120.
D 6	5|580		D, \$122.
E 8	——		E, \$124.
——	116		
20			

36. Divide £20 among six men, giving B 8 pence more than A, C 8 d. more than B, D 8 d. more than C, &c.

Ans. A, £3 5 s.; B, £3 5 s. 8 d.; C, £3 6 s. 4 d.; D, £3 7 s.; E, £3 7 s. 8 d.; F, £3 8 s. 4 d.

Note.—When the ratios are not equal, the same rule may be used.

37. Divide \$1600 among 5 men. Give B \$5 more than A, C \$7 more than B, D \$12 more than C, E \$35 more than D. *Ans.* A, \$300, B, 305, C, 312, D, 324, E, 359.

38. Divide \$1 by 4 cents.

\$1=100 cents; then 100÷4=25, which shows that 4 cents are contained 25 times in \$1.

39. Divide £1 by 1 shilling.

£1=20 s.; then 20÷1=20, which shows that 1 shilling is contained 20 times in £1.

40. Divide 3 bushels by 3 quarts.

3 bushels=96 quarts; then, 96÷3=32, which shows that 3 quarts are contained 32 times in 3 bushels.

41. Divide 4 acres by 5 rods.

4 acres=640 rods; then, 640÷5=128, which shows that 5 rods are contained 128 times in 4 acres.

Note.—The four preceding questions are inserted to expose an error in Waterhouse's Key, which makes the answers in the above sums to be \$25, £20, 32 bushels, and 128 acres. On examining his rule, page 15, it will be seen he entirely mistakes the nature of the subject.

42. From 14 years subtract 11 years, 11 months, 11 weeks, 11 days, 11 hours, 11 minutes, 11 seconds.

		13	4	7	24	60	60
	14	0	0	0	0	0	0
	11	11	11	11	11	11	11
Ans.	1	11	3	2	12	48	49

The intelligent learner will see that this problem is not given in a regular mathematical order, and is wrought in this manner by borrowing to suit the circumstances of the case; and as 12 cannot be taken from 7, double the 7 (7×2) $=14$; then $14-12=2$, and as we have borrowed 2, carry 2 to 11, making 13; then, as 13 cannot be taken from 4, 8, nor 12, borrow 4 times $4=16$; then $16-13=3$, carrying 4 to the next $11+4=15$; then $13\times2=26-15=11$; carry 2 to $11=13$; then $14-13=1$, which gives the true answer.

43. Suppose a man hires a boy to serve him 10 years, and the lad stays 9 years, 9 months, 9 weeks, 9 days, 9 hours, 9 minutes, and 9 seconds; how much longer should he have staid to have fulfilled the contract?

Ans. 1 m. 1 w. 4 d. 14 h. 50 min. 51 sec.

Note.—The same method may be observed in solving all problems given in a similar manner.

44. What must be the length of a piece of land that is 3 rods in width to make an acre? (See page 131.)

Ans. $53\frac{1}{3}$ rods.

45. What must be the length of a piece of land that is 12,5 rods in width to make an acre? *Ans.* 12,8 rods.

If $16\frac{2}{3}$ wide, what must be the length? *Ans.* 9,6 rods.

46. Suppose A and B to take 100 rods of stone wall to build for $100, and on examination they find that one end of the wall is worth $1,25 per rod, and they agree that A shall lay $50 dollars worth on one end at that price, and B is to finish the wall for the other $50. How much must B lay of it, and what must be his price per rod?

Ans. B lays 60 rods at 5 s.=83\frac{1}{3}$ cts. per rod.

47. What must be the length of a stock of hewn timber that is 10 inches wide, and 1 foot 3 inches deep, in order to contain 40 feet? (See p. 132.) *Ans.* 460,8 in.=38,4 ft.

48. Suppose wood to be piled on a base 15 feet, 6 inches long, and 7 feet, 9 inches wide; what must be the height of the pile to contain 16 cords? *Ans.* $17\frac{943}{4761}$ feet.

49. Divide 360 into 4 such parts as shall be to each other as 3, 4, 5, 6. *Ans.* A 60, B 80, C 100, D 120.

50. Suppose a man to be hired for 79,5 days, at $1 per day for every day he works, and for every day he is idle he is to forfeit 25 cents, and he receives for pay $67,75. How many days has he worked, and how many was he idle?
Ans. He worked 70,1 days; was idle 9,4 days.

51. Divide $,125 between 2 boys in such a proportion, that after it is divided, one shall have $\frac{1}{3}$ more than the other. What is the share of each? *Ans.* A has ,075, *B,05.*

52. In eleven hundred barrels of beef, each weighing two hundred, how many hundred weight?

53. What difference is there between twice five and twenty, and twice twenty-five? *Ans.* 20.

54. A was born when B was 21 years of age; how old will A be when B is 47, and what will be the age of B when A is 60? *Ans.* A 26, B 81.

55. Two persons depart from the same place, at the same time; one travels 30, the other 25 miles a day; how far apart are they at the end of 7 days, if they both travel the same way? and how far, if they travel in contrary directions? *Ans.* 35 miles, and 455 miles.

56. A, B, and C are to share $100000 in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ respectively; but B's part being lost by his death, it is required to divide the whole sum properly between the other two. What is the share of each?
Ans. A's part, $$57142,85\frac{5}{7}$; B's, $42857,14\frac{2}{7}$.

57. A gentleman, on being asked the time of day, replied, that $\frac{2}{3}$ of the time since noon equals $\frac{8}{42}$ of the time from now till midnight; what o'clock was it?
Ans. 40 minutes past 1 o'clock.

58. Suppose 3 men are employed about a certain piece of labor for $238,05, and A and B are supposed to do $\frac{3}{4}$ of the work, A and C $\frac{9}{10}$, and B and C $\frac{13}{20}$, and they are to be paid proportionally. What is the share of each?

Ans. A, $103,5; B, $82,8; C, $51,75.

59. Four men trade together on a capital of $6487, of which A put in $\frac{1}{2}$, B $\frac{1}{4}$, C $\frac{1}{6}$, and D $\frac{1}{12}$. At the end of 5 years they had gained $4372; what was the share of each?

Ans. A $2186, B $1093, C $728,66$\frac{2}{3}$, D 364,33$\frac{1}{3}$.

60. Four merchants accompanied; A put in $\frac{2}{5}$, B $\frac{3}{16}$, C $\frac{7}{32}$, and D the remainder, and they gained $1428,57$\frac{1}{7}$. How much did D put in, and what was the share of each?

Ans. D put in ,19375. A's share, $571,42$\frac{6}{7}$; B's, $267,85$\frac{5}{7}$; C's, $312,5; D's, $276,78$\frac{4}{7}$.

61. A farmer, being asked how many sheep he had, answered that he had them in five fields; in the first were $\frac{1}{4}$ of his flock, in the second $\frac{1}{6}$, in the third $\frac{1}{8}$, in the fourth $\frac{1}{12}$, and in the fifth 450; how many had he? *Ans.* 1200.

62. The third part of an army was killed, the fourth part taken prisoners, and 1000 fled; how many were in this army? *Ans.* 2400 men.

63. There is a pole, $\frac{1}{4}$ of which stands in the mud, $\frac{1}{3}$ in the water, and the rest of it out of the water; required the part out of the water. *Ans.* $\frac{5}{12}$.

64. If a pole be $\frac{1}{3}$ in the mud, $\frac{3}{5}$ in the water, and 6 feet out of the water, what is the length of the pole? *Ans.* 90 ft.

65. The amount of a certain school is as follows: $\frac{1}{16}$ of the pupils study grammar, $\frac{3}{8}$ geography, $\frac{3}{10}$ arithmetic, $\frac{3}{20}$ learn to write, and 9 learn to read; what is the number of each?

Ans. 5 in grammar, 30 in geography, 24 in arithmetic; 12 learn to write, and 9 learn to read.

66. Suppose a cent to be 1 inch in diameter, how many cents will it take to be so laid as to enclose a square piece of ground containing as many acres as there are cents, and to what extent of distance?

Ans. 1584 miles, 100362240 cts. 100362240 acres.

FORMS OF NOTES, BONDS, AND RECEIPTS.

NOTES.

No. I.

Boston, January 1, 1842.

For value received, I promise to pay Oliver Bountiful, or order, sixty-three dollars fifty-four cents, on demand, with interest after three months. Timothy Trustall.

Attest, Peter Lookout.

No. II.

Concord, February 2, 1842.

For value received, I promise to pay A. B., or bearer, twenty-one dollars ten cents, in three months after date. Peter Pepper.

No. III.—*By two Persons.*

Peterboro', February 21, 1842.

For value received, we jointly and severally promise to pay L. A. Fletcher, or order, one hundred seven dollars twenty cents, on demand, with interest.

Peter Simple.
Jacob Faithful.

Attest, Thomas Adams.

OBSERVATIONS.

1. No note is negotiable unless the words "*or order,*" otherwise "*or bearer,*" be inserted in it.

2. If the note be written to pay him "*or order,*" (No. I.) then Oliver Bountiful may endorse this note, that is, write his name on the back of it, and sell it to A, B, C, or whom he pleases. Then A, who buys the note, calls on Timothy Trustall for payment, and if he neglects, or is unable to pay, A may recover it of the endorser.

3. If a note be written to pay him "*or bearer,*" (No. II.) then any person who holds the note may sue and recover the same of Peter Pepper.

4. The rate of interest, established by law, being *six per cent per annum*, it becomes unnecessary, in writing notes, to mention the rate of interest; it is sufficient to write them for the payment of such a sum, with interest, for it will be understood legal interest, which is six per cent.

5. All notes are either payable on demand, or at the expiration of a certain term of time agreed upon by the parties, and mentioned in the note; as, three months, a year, &c.

6. If a bond or note mention no time of payment, it is always on demand, whether the words "*on demand*" be expressed or not.

7. All notes, payable at a certain time, are on interest as soon as they become due, though in such notes there be no mention made of interest.

This rule is founded on the principle that every man ought to receive his money when due, and that the non-payment of it at that

time is an injury to him. The law, therefore, to do him justice, allows him interest from the time the money becomes due, as a compensation for the injury.

8. Upon the same principle, a note, payable on demand, without any mention made of interest, is on interest after a demand of payment; for upon demand such notes immediately become due.

9. If a note be given for a specific article, payable in one, two, or three months, or in any certain time, and the signer of such note suffers the time to elapse without delivering such article, the holder of the note will not be obliged to take the article afterwards, but may demand and recover the value of it in money.

BONDS.

A Bond, with a Condition, from one to another.

Know all men by these presents, that I, C— D— of —— in the county of ——, &c., am held and firmly bound to C— D— of, &c., in two hundred dollars, to be paid to the said E— F—, or to his certain attorney, his executors, administrators, or assigns, to which payment, well and truly to be made, I bind myself, my heirs, executors and administrators, firmly by these presents. Sealed with my seal. Dated the eleventh day of —— in the year of our Lord one thousand eight hundred and forty-two.

The condition of this obligation is such, that if the above-bound C— D—, his heirs, executors, administrators, or assigns, do and shall well and truly pay, or cause to be paid, unto the above-named E— F—, his executors, administrators, or assigns, the full sum of two hundred dollars, with legal interest for the same, on or before the eleventh day of —— next ensuing the date hereof,—then this obligation to be void, or otherwise to remain in full force and virtue.

Signed, &c.

A Condition of a Counter Bond, or Bond of Indemnity, where one Man becomes bound for another.

The condition of this obligation is such, that whereas the above-named A— B—, at the special instance and request, and for the only proper debt of the above-bound C— D—, together with the said C— D—, is, and by one bond or obligation bearing equal date with the obligation above written, held and firmly bound unto E— F— of, &c., in the penal sum of —— dollars, conditioned for the payment of the sum of —— with legal interest for the same, on the —— day of —— next ensuing the date of the said in part recited obligation, as in and by the said in part recited bond, with the condition thereunder written, may more fully appear;—if, therefore, the said C— D—, his heirs, executors, or administrators, do and shall well and truly pay, or cause to be paid, unto the said E— F—, his executors, administrators, or assigns, the said sum of —— dollars, with legal interest on the same, on the said —— day of —— next ensuing the date of the said in part recited obligation, accord-

ing to the true intent and meaning, and in full discharge and satisfaction of the said in part recited obligation,—then, &c.—otherwise, &c.

Note.—The principal difference between a note and a bond is, that the latter is an instrument of more solemnity, being given under seal. Also, a note may be controlled by a special agreement, different from the note; whereas, in case of a bond no special agreement can in the least control what appears to have been the intention of the parties, as expressed by the words in the condition of the bond.

RECEIPTS.

Boston, February 15, 1842.

Received of Mr. PETER PAYWELL ten dollars in full of all accounts. JOHN DUNN.

Boston, February 15, 1842.

Received of Mr. JOHN DUNN five dollars in full of all accounts. PETER PAYWELL.

Receipt for Money received on a Note.

Concord, January 1, 1842.

Received of Mr. JACOB FAITHFUL (by the hand of Titus Trusty) sixteen dollars eighty-five cents, which is endorsed on his note of July 4, 1839. JOHN PUNCTUAL.

Receipt for Money received on Account.

Windsor, February 14, 1842.

Received of Mr. JOHN SLACK fifty dollars on account. WILLIAM HOLDFAST.

Receipt for Interest due on a Note.

Montpelier, January 1, 1842.

Received of Mr. JOHN TRUSTY thirty dollars, in full of one year's interest of $500, due to me on the 29th day of December last, on note from said John Trusty. PETER TRUMAN.

Note.—There is a distinction between receipts given in full of *all accounts*, and others in full of *all demands*. The former cut off accounts *only;* the latter cut off not only accounts, but all obligations and right of action.